AERA 2015 Annual Meeting, 16 April 2015
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On A Learning Trajectory for Reciprocal Reasoning with Quantitative Unknowns

Fern-Sunflower Height Problem. A fern and sunflower are growing in the garden, each of unknown height. The height of the sunflower is 3/5 the height of the fern.

a. Draw a picture of this situation and describe what your picture represents
b. Write an equation for this situation that relates the two heights. Explain what your equation means in terms of your picture.
c. Can you write another, different equation that relates the two heights? Explain what your equation means in terms of your picture.

Data Except 1: Gabriel_8, Stephanie_7, and the teacher talk about the equations on 10/10/13.

T: I'm especially interested in this [the 5/3y = x equation], but why don't we start here [with the 3/5x = y equation]. How do you see this in your picture?
G: Well, three-fifths, is right here [pointing to the 3-part segment in his drawing, cf. Figure 1]. Three-fifths of x equals 1 y. So I just lined it up here [in his drawing] to show that and I also put the 1 here [in front of the y].
T: That also makes sense to you Stephanie?
[S nods.]
T: Okay, but now how on earth do you know that [5/3y = x]?
G: Well because multiplying it by its reciprocal will do that, and I just know that.
T: But in your picture,
S: I didn't know that.
G [to T, as if he knows he needs to explain more]: Yeah I know.
T: How could you justify it in your picture? That's what I want to know.
G: Hmm. [He gives small laugh, as if uncertain.]
T: So think about that. That's the key thing to think about.

Data Excerpt 2: Gabriel_8 and Martin_7 switch the referent unit on 10/10/13.

T: Okay, but that's what I want to know how you know.
M: Well, since it's the reciprocal.
T: Yeah, but I want you to be able to justify it in your picture. Justify to me why that works.
G: Impossible!
M: Oh you could do this, divide it into thirds. Instead of—
T: Divide what into thirds?
G: Oh yeah! You could divide this whole thing into thirds and then add two extra thirds and it equals this. Oh my gosh!
M: Yeah five-thirds is—
[Laughter; boys high five each other.]
T: So when you say, divide this whole thing into thirds, what did you mean by this whole thing?
G: The, um, sunflower.

Data Excerpt 3: Gabriel_8 expresses uncertainty about the equality on 10/10/13.

G: Well I mean, it may or may not be equal.
T: Oh, you don't think it's equal?
G: Well I'm not saying it isn't.
M: No, it is; it *is* equal.
T [to G]: Because we don't know the values? If this relationship holds, though, then it should be equal or no?
G: Well I mean it just lines up, and I'm a horrible artist. So.

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The Heights A & B Problem. Let's say A represents the height of one object, and B represents the height of another object. If you know that A is 2/7 of B, explain how you can determine what fraction B is of A. Use diagrams to help you explain.

The Weights C & D Problem. Let's say C represents the weight of one object, and D represents the weight of another object. If you know that C is 7/5 of D, explain how you can determine what fraction D is of C. Use diagrams to help you explain.

Two Unknowns X & Y Problem. Does the reasoning you explained in The Heights A & B Problem and the Weights C & D Problem apply to any fractional relationship between two quantities? For example, if X and Y represent unknowns, and X is 13/27 of Y, can you apply the reasoning in the two prior problem to determine what fraction Y is of X?

Pine Tree Problem. Steven and Lia each are growing a pine tree. The height of each of their trees is unknown. The height of Lia's plant is 2/5 the height of Steven's plant.

a. Draw a picture of the situation and describe what your picture represents.
b. Write an equation for this situation that relates the two heights. Explain your equation in terms of your picture.
c. Can you write another, different equation that relates the two heights? Explain this equation in terms of your picture.

Revised Heights A & B Problem. Let's say A represents the height of one object, and B represents the height of another object.

a. If you know that A is 2/7 of B, draw a picture and explain how you can determine what fraction B is of A. You can use JavaBars.
b. Sometimes people write an equation like this to relate A and B: A + 5/7 = B. Will that equation work? Explain and tell what this equation means in the picture.
c. Sometimes people write an equation like this to relate A and B: A + A + A + 1/2A = B. Will that equation work? Explain and tell what this equation means in the picture.
d. Sometimes people write an equation like A ÷ 2 x 7 = B. Will that equation work? Explain and tell what this equation means in the picture.
e. The point of this problem is to think about how to communicate ideas with algebraic notation. Sometimes people don't believe that **2/7 of a quantity B** can be written with multiplication as **(2/7)*B**.
   Do you believe that **2/7 of a quantity B** can be written with multiplication as **(2/7)*B**? Please be honest. Circle YES or NO
   If you do believe it, do you have a way to explain or justify? Please tell:

Revised Two Unknowns X & Y Problem. Let's say that X and Y represent unknown heights, and X is 13/41 of Y. Can you use the reasoning from prior problems to determine what fraction Y is of X? **Explain your reasoning.** Please do not just say that you use the reciprocal. Explain how the reciprocal comes about by thinking about the quantities.

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