ON A LEARNING TRAJECTORY FOR RECIPROCAL REASONING WITH QUANTITATIVE UNKNOWNS

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Rationale

- Students continue to have difficulty conceiving of and operating with rational numbers (Kerslake, 1986; Lamon, 2007; Wearne & Hiebert, 1988), and they have difficulty passing algebra courses (Helfand, 2006; Stein, Kaufman, Sherman, & Hillen, 2011)
- Learning trajectories are needed in algebra (Daro, Mosher, & Corcoran, 2011).
- And, to our knowledge, no learning trajectories have been developed to connect key domains like rational number and algebra.

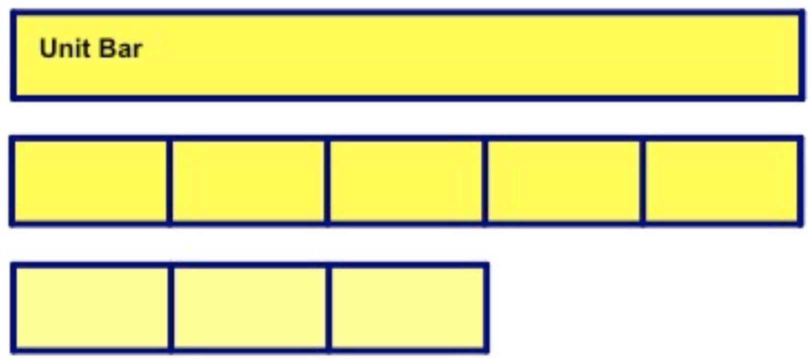
Purpose

- Articulate a learning trajectory for reciprocal reasoning with quantitative unknowns for middle school students
  - Data come from two 18-session design experiments, each with nine 7^{th} and 8^{th} grade students
  - 3 students in each experiment constructed RR with quantitative unknowns, and they made accommodations in their fractional knowledge to do so

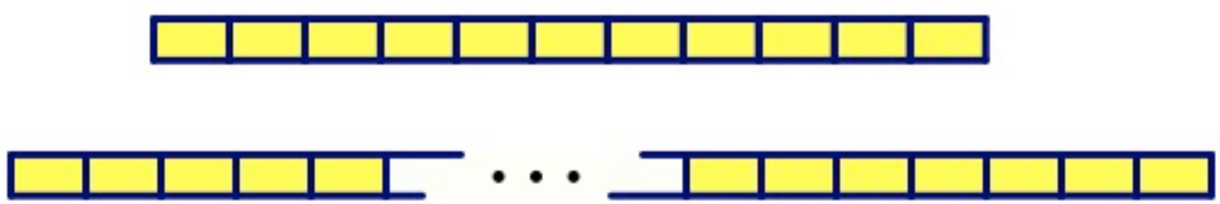
- Elements of a learning trajectory (Steffe, 2004)
  - our models of the students’ ways of operating
  - changes we experienced in students’ ways of operating
  - mathematical interactions involved in the changes, including social interactions

A Quantitative Approach

- Fractions as quantities

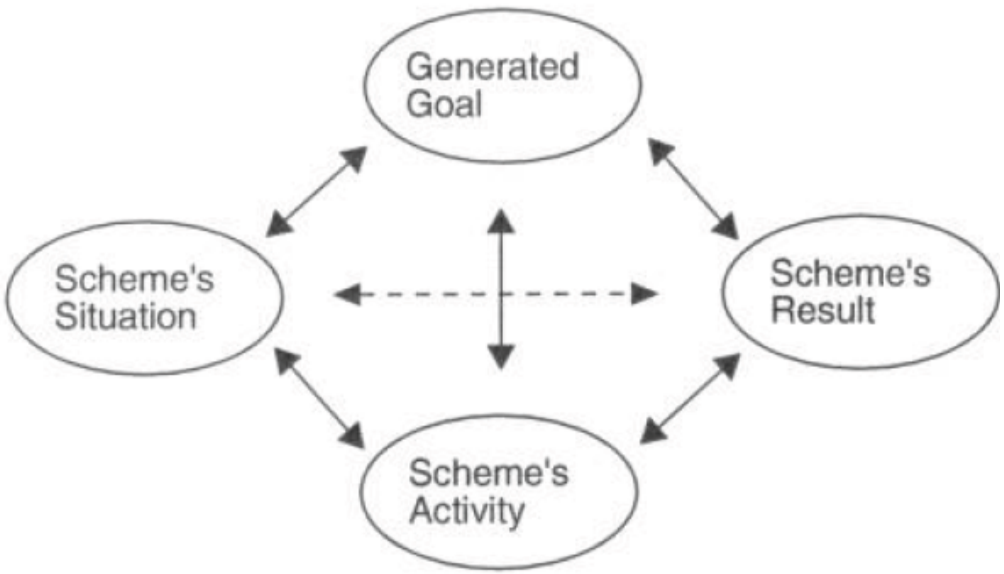


- Unknowns as potential measurements of quantities



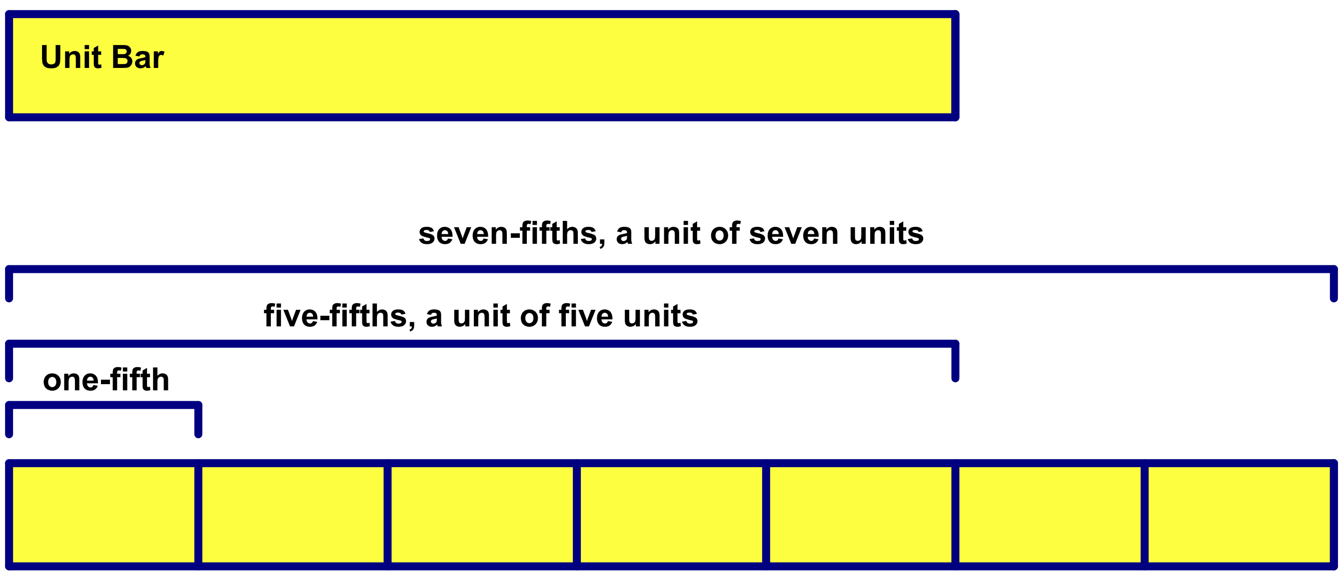
Learning and Stable Ways of Knowing

- Learning: Making accommodations in schemes in on-going interaction in one's experiential world
  - Schemes: goal-directed ways of operating that involve an assimilated situation, an activity, and a result that a learner assesses in relation to her/his goals (Steffe & Olive, 2010; von Glasersfeld, 1995)
- Stable ways of knowing:
  - Participatory concepts v. anticipatory concepts (Tzur & Simon, 2004)
  - Searching…



Iterative Fraction Scheme

- Fractions are whole number multiples of unit fractions



Method

- **Two 18-episode design experiments conducted after school**
  - 6 MC3 students in 7th and 8th grade (3 out of 9 in each experiment);
  - 1-hr episodes twice per week for 9 weeks
- **Selection process:** interview and written worksheet with 21-24 students to assess multiplicative concepts, fractional knowledge
- **During the episodes:**
  - Flow of whole class discussion/activity and small group work
  - One researcher was the teacher; other researchers assisted by running roaming cameras, interacting with students, taking notes
- **Data collection included:**
  - Video recording all episodes with 3 cameras (mixed into 1 file for analysis)
  - Screenflow recordings of students' work in small groups on laptops; students used JavaBars and The Geometer's Sketchpad
- **Data analysis:** Underway!
  - Student portraits

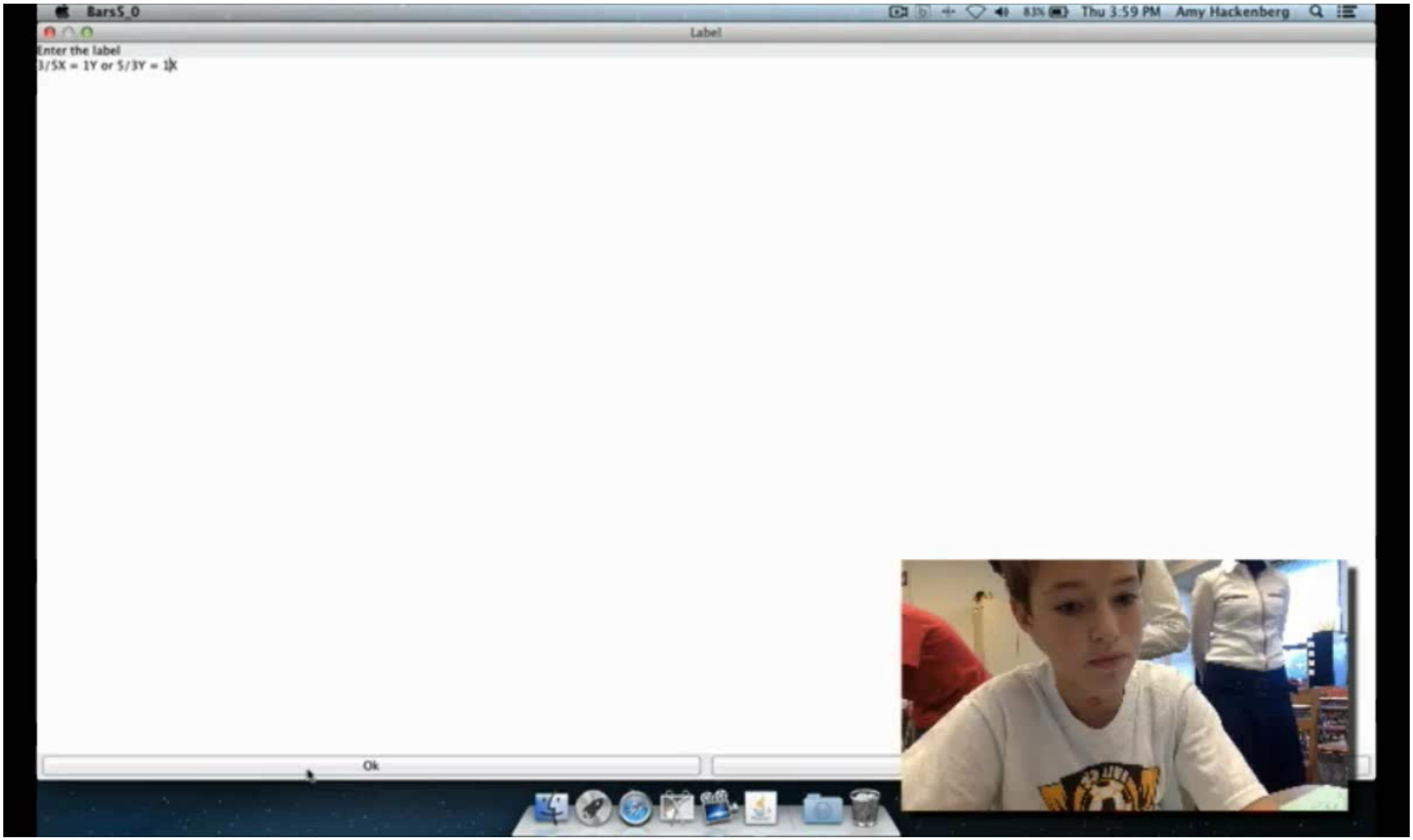
MC3 Participants

Name	Grade	Course Enrollment	Participation Semester
Gabriel	8th	Algebra	Fall
Martin	7th	Adv. 7th gr. math	Fall
Stephanie	7th	Adv. 7th gr. math	Fall
Brad	7th	Algebra	Spring
Amanda	7th	Adv. 7th gr. math	Spring
Harry	7th	Reg. 7th gr. math	Spring

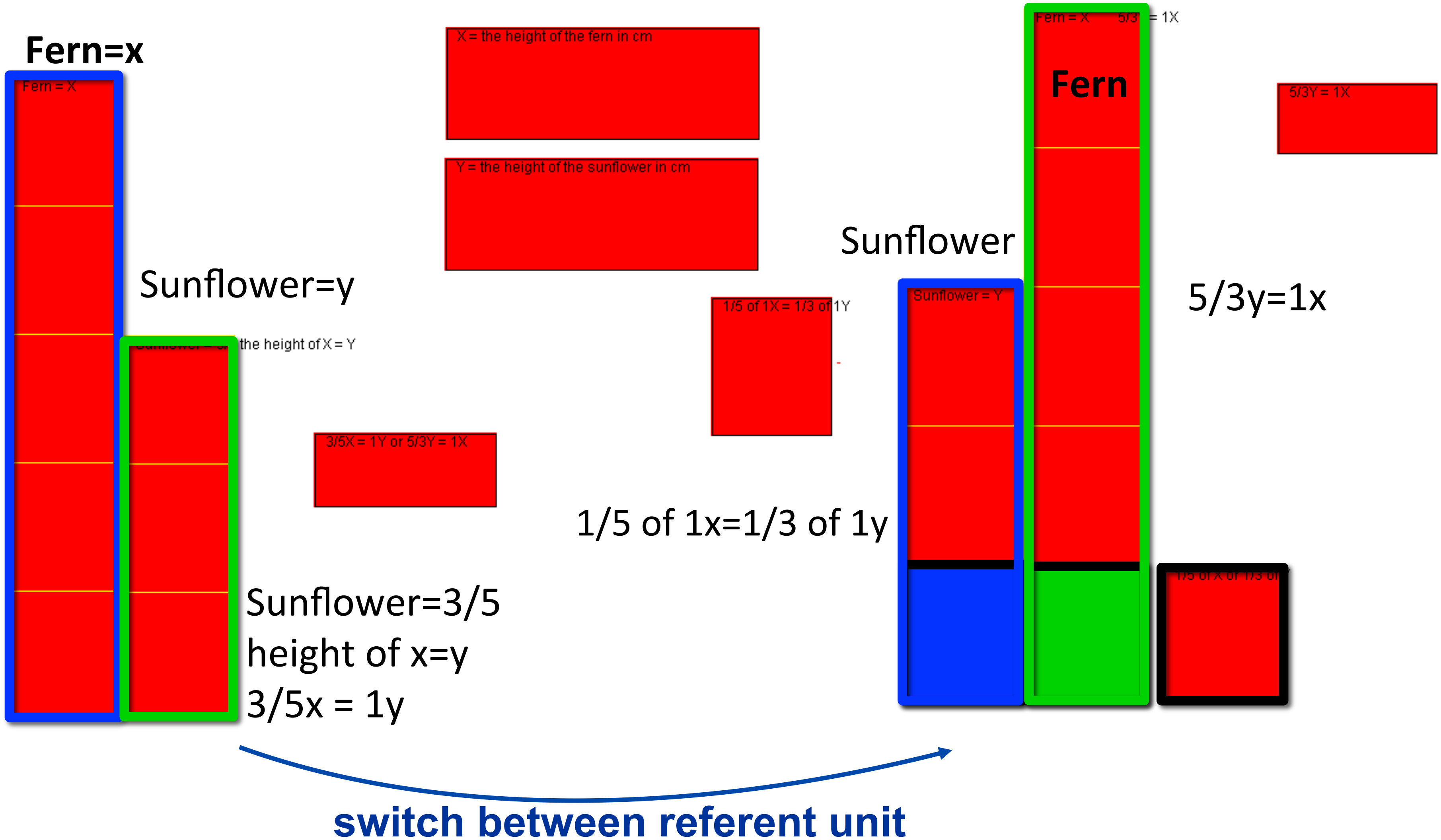
Fern-Sunflower Heights Problem

- A fern and sunflower are growing in the garden, each of unknown height. The height of the sunflower is 3/5 the height of the fern.
  - Draw a picture of this situation and describe what your picture represents.
  - Write an equation for this situation that relates the two heights. Explain your equation in terms of your picture.
  - Can you write another, different equation that relates the two heights? Explain this equation in terms of your picture.

Martin (7th) and Gabriel (8th)



Account of Switching Referent Unit



Next Steps in fall 2013

- **The Heights A & B Problem.** Let's say A represents the height of one object, and B represents the height of another object. If you know that A is 2/7 of B, explain how you can determine what fraction B is of A. Use diagrams to help you explain.
- **The Weights C & D Problem.** Let's say C represents the weight of one object, and D represents the weight of another object. If you know that C is 7/5 of D, explain how you can determine what fraction D is of C. Use diagrams to help you explain.
- **Two Unknowns X & Y Problem.** Does the reasoning you explained in The Heights A & B Problem and the Weights C & D Problem apply to any fractional relationship between two quantities? For example, if X and Y represent unknowns, and X is 13/27 of Y, can you apply the reasoning in the two prior problem to determine what fraction Y is of X?

Key Components of Interaction for the construction of RR with QU

- **Initial task design:**
  - Initial task set in quantitative situation and included a request to draw picture
- **Teacher-student interaction during the tasks:**
  - Press for justification of equations in pictures
- **Student-student interaction during the tasks:**
  - Encouragement to voice their ideas to each other in small group and whole class discussion
- **Follow-up task design:**
  - Attempts to provide repeated experiences for students in order to develop stable or generalized knowing

Revisions in spring 2015 (one example)

- **Revised Heights A & B Problem.** Let's say A represents the height of one object, and B represents the height of another object.
  - If you know that A is 2/7 of B, draw a picture and explain how you can determine what fraction B is of A. You can use JavaBars.
  - Sometimes people write an equation like this to relate A and B: A + 5/7 = B. Will that equation work? Explain and tell what this equation means in the picture.
  - Sometimes people write an equation like this to relate A and B: A + A + A + 1/2A = B. Will that equation work? Explain and tell what this equation means in the picture.
  - Sometimes people write an equation like A ÷ 2 x 7 = B. Will that equation work? Explain and tell what this equation means in the picture.
  - The point of this problem is to think about how to communicate ideas with algebraic notation. Sometimes people don't believe that **2/7 of a quantity B** can be written with multiplication as **(2/7)*B**.
    Do you believe that **2/7 of a quantity B** can be written with multiplication as **(2/7)*B**? Please be honest. Circle YES or NO
    If you do believe it, do you have a way to explain or justify? Please tell:

Thank you!

IDR^{2}eAM stands for **I**nvestigating **D**ifferentiated Instruction and **R**elationships between **R**ational Number Knowl**e**dge and **A**lgebraic Reasoning in **M**iddle School.

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