Tiering Instruction on Speed for Middle School Students

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While you wait, check out the Geogebra app “NewRace” by Janet Bowers. Search for NewRace on geogebra.org
Two-prong Rationale

- **Classrooms are increasingly diverse** (National Center of Educational Statistics, 2016)
- **Differentiating instruction: a pedagogical approach to manage classroom diversity**
  - An alternative to tracking
  - Secondary math classrooms are places differentiation is least likely to occur (Gamoran & Weinstein, 1998)
- **Proportional reasoning is a BIG, challenging idea in middle school** (Kaput & West, 1994; Lamon, 2007; Lesh, Post, & Behr, 1988)
- **Studying speed has been used to support students’ proportional reasoning** (Ellis, 2007; Lobato & Siebert, 2002)
  - Hallmark of the construction of speed: “partitioning a traveled total distance implies a proportional partition of total time required to travel that distance” and vice-versa (Thompson & Thompson, 1994, p. 283).
Research Question

- What influences did tiering instruction with speed tasks have on a class of 18 regular seventh grade mathematics students during a unit on proportional reasoning?

- **Differentiating Instruction:** proactively tailoring instruction to students’ mathematical thinking while developing a cohesive classroom community (Hackenberg, Creager, & Eker, under review)

- **Tiering Instruction:** designing different problems (or sequences of problems) for different groups of students in a classroom based on conjectures about what will support students’ learning (Tomlinson, 2005).
Students’ Multiplicative Concepts

- **Unit**: discrete one, length, standard or non-standard measurement unit
- **Composite unit**: a unit of units
- **Units coordination**: distribute or insert the units of one composite unit across the units of another composite unit
MC1, MC2, and MC3 students

MC1

MC2

MC3
Research on Multiplicative Concepts

- **Relatively stable**, e.g., 2 years (Steffe & Cobb, 1988; Steffe, 2017)
- **Influences:**
  - Fractions knowledge (Steffe & Olive, 2010)
  - Algebraic reasoning (Hackenberg & Lee, 2015; Olive & Caglayan, 2008)
  - Integers (Ulrich, 2012)
  - Combinatorial reasoning (Tillema, 2013)
- **Estimates for in-coming 6th grade students:** 30% MC1, 30% MC2, 40% MC3 (Steffe, 2017)
Method - Participant Selection

- Participating classroom: 7th grade pre-algebra, 18 students
- Selected one other classroom for comparison: 20 students
- Gathered initial written assessments and individual interviews
- Results:

<table>
<thead>
<tr>
<th>Units Coordination Level</th>
<th>Participating Class</th>
<th>Comparison Class</th>
</tr>
</thead>
<tbody>
<tr>
<td>MC1</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>MC2</td>
<td>9</td>
<td>8</td>
</tr>
<tr>
<td>MC3</td>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>

- Selected 6 participating focus students (two MC1, three MC2, one MC1) and 6 comparison focus students matched on units coordination and aspects of fraction knowledge.
Method - Data Collection and Analysis

Students worked on Comparing and Scaling, a 7th grade CMP unit focused on ratios and proportional reasoning.

Data:
- Daily: whole-class and small group video, copies of written student work
- Middle of Unit - 6 focus students from participating class
- End of the Unit - 12 focus students - participating and comparison

Analysis:
Development of second-order models of student thinking - analysis of all data sources, discussions with research team
Summary of Days 9-13

Days 9 - 10

Days 11 - 13

SAME SPEED TASK: The blue car travels _____ miles in _____ minutes. Make the red car travel at the same speed as the blue car, but the red car will travel a different amount of miles and a different amount of minutes.

Tiering plan

<table>
<thead>
<tr>
<th>Mult. Concept</th>
<th>Orangeyness Investigation</th>
<th>Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>MC1</td>
<td>*Not fluidly iterating two quantities as a composed unit.</td>
<td>18 mi in 3 min *Whole number unit ratio (6 mi in 1 min)</td>
</tr>
<tr>
<td>MC2</td>
<td>*Iterating two quantities as a composed unit.</td>
<td>15 mi in 6 min *Mixed number unit ratio with ½ (2.5 mi in 1 min)</td>
</tr>
<tr>
<td>MC3</td>
<td>*Iterating two quantities as a composed unit *Making unit ratios</td>
<td>15 mi in 9 min *Unit ratio hard to work with as a decimal (5/3 mi in 1 min)</td>
</tr>
</tbody>
</table>
Emily and group (MC1 students)
18 miles in 3 min
Emily and group (MC1): 2\textsuperscript{nd} picture
18 miles in 3 min
Emily’s pictures: 18 miles in 3 min
Lisa and Sara (MC2): breakthrough
15 miles in 6 min
Lisa and Sara (MC2): picture development
15 miles in 6 min
Lisa’s pictures: 15 miles in 6 min
Joanna (MC3)
15 miles in 9 min
“I was showing how much the blue car went, which is 15 miles in 9 minutes.”

“I divided up into three, so it’s 0, then it goes to 5 and 10 and 15. And this goes 0, 3 to 6 to 9.”

”They go the same speed, but this one [red car] just stops earlier.”

”5 miles in 3 minutes would be one third of the trip.”
Student Learning

Emily

- Saw that doubling both distance and time “worked”
- Did not have a way to show doubling with pictures
- Created a doubled journey as two smaller same-size journeys, with support
- Needed support to articulate how to justify same speeds

Lisa

- Saw that doubling both distance and time “worked”
- Showed doubling in her picture right away
- Created a doubled journey as two smaller same-size journeys
- Needed support to articulate how to justify same speeds
Student Learning

Lisa & Sara

Joanna
Lisa and Sara continued...
Self-assessment of Tiering: Were the number choices good ones?

- Emily: Yes
- Lisa and Sara: Yes, but...
- Joanna: Yes
This study shows how differentiating instruction supported student learning of reasoning with ratios across multiplicative concepts.

We argue that this is generally better for students than “one-size-fits-all” instruction.

Differentiation is touted as a component of inclusive mathematics classrooms in which equity is a priority (e.g., Boaler, 2019; Michael, 2015), but:

- Differentiating is hard to do!
- Need curricular materials
- Need professional development
THANK YOU!

- With BIG thanks to all other members of the IDR²eAM project team, past and present: Rebecca Borowski, Mark Creager, Anna Dinndorf, Ayfer Eker, Sharon Hoffman, Robin Jones, Musa Sadak, Serife Sevinc, Pai Suksak, Ryan Timmons, and Erol Uzan

- **Special thanks** to middle school teachers Patti Walsh and Marie Johannisson

- **What IDR²eAM stands for:**
  - Investigating Differentiated Instruction and Relationships between Rational Number Knowledge and Algebraic Reasoning in Middle School

- [http://www.indiana.edu/~idream/](http://www.indiana.edu/~idream/)