While you wait, check out the Geogebra app "NewRace" by Janet Bowers. Search for NewRace on geogebra.org



Tiering Instruction on Speed for Middle School Students

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Two-prong Rationale

- Classrooms are increasingly diverse (National Center of Educational Statistics, 2016)
- Differentiating instruction: a pedagogical approach to manage classroom diversity
 - An alternative to tracking
 - Secondary math classrooms are places differentiation is least likely to occur (Gamoran & Weinstein, 1998)
- Proportional reasoning is a BIG, challenging idea in middle school (Kaput & West, 1994; Lamon, 2007; Lesh, Post, & Behr, 1988)
- Studying speed has been used to support students' proportional reasoning (Ellis, 2007; Lobato & Siebert, 2002)
 - Hallmark of the construction of speed: "partitioning a traveled total distance implies a proportional partition of total time required to travel that distance" and vice-versa (Thompson & Thompson, 1994, p. 283).

Research Question

- What influences did tiering instruction with speed tasks have on a class of 18 regular seventh grade mathematics students during a unit on proportional reasoning?
- Differentiating Instruction: proactively tailoring instruction to students' mathematical thinking while developing a cohesive classroom community (Hackenberg, Creager, & Eker, under review)
 - Tiering Instruction: designing different problems (or sequences of problems) for different groups of students in a classroom based on conjectures about what will support students' learning (Tomlinson, 2005).

Students' Multiplicative Concepts

- **Unit:** discrete one, length, standard or non-standard measurement unit
- Composite unit: a unit of units
- Units coordination: distribute or insert the units of one composite unit across the units of another composite unit





Research on Multiplicative Concepts

Relatively stable, e.g., 2 years (Steffe & Cobb, 1988; Steffe, 2017)

Influences:

- Fractions knowledge (Steffe & Olive, 2010)
- Algebraic reasoning (Hackenberg & Lee, 2015; Olive & Caglayan, 2008)
- Integers (Ulrich, 2012)
- Combinatorial reasoning (Tillema, 2013)
- Estimates for in-coming 6th grade students: 30% MC1, 30% MC2, 40% MC3 (Steffe, 2017)

Method - Participant Selection

- > Participating classroom: 7th grade pre-algebra, 18 students
- Selected one other classroom for comparison: 20 students
- Gathered initial written assessments and individual interviews
- Results:

Units Coordination Level	Participating Class	Comparison Class
MC1	5	6
MC2	9	8
MC3	4	6

Selected 6 participating focus students (two MC1, three MC2, one MC1) and 6 comparison focus students matched on units coordination and aspects of fraction knowledge

Method - Data Collection and Analysis

Students worked on Comparing and Scaling, a 7th grade CMP unit focused on ratios and proportional reasoning.

Data:

- Daily: whole-class and small group video, copies of written student work
- Middle of Unit 6 focus students from participating class
- End of the Unit 12 focus students participating and comparison

Analysis:

Development of second-order models of student thinking analysis of all data sources, discussions with research team

Summary of Days 9-13

- Days 9 10
- Days 11 13
 - SAME SPEED TASK: The blue car travels _____ miles in _____ minutes. Make the red car travel at the <u>same</u> speed as the blue car, but the red car will travel a different amount of miles and a different amount of minutes.
- Tiering plan



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Mult. Concept	Orangeyness Investigation	Numbers
MC1	*Not fluidly iterating two quantities as a composed unit.	18 mi in 3 min *Whole number unit ratio (6 mi in 1 min)
MC2	*Iterating two quantities as a composed unit.	15 mi in 6 min *Mixed number unit ratio with $\frac{1}{2}$ (2.5 mi in 1 min)
MC3	*Iterating two quantities as a composed unit *Making unit ratios	15 mi in 9 min *Unit ratio hard to work with as a decimal (5/3 mi in 1 min)

Emily and group (MC1 students) 18 miles in 3 min



Emily and group (MC1): 2nd picture 18 miles in 3 min



Emily's pictures: 18 miles in 3 min





<u>18 miles</u> 3 mih. Km.1e Car 3 mir Semiles Far 6 min.

Lisa and Sara (MC2): breakthrough 15 miles in 6 min



Lisa and Sara (MC2): picture development 15 miles in 6 min



Lisa's pictures: 15 miles in 6 min





Joanna (MC3) 15 miles in 9 min



Joanna's picture: 15 miles in 9 min



- "I was showing how much the blue car went, which is 15 miles in 9 minutes."
- "I divided up into three, so it's 0, then it goes to 5 and 10 and 15. And this goes 0, 3 to 6 to 9."
- They go the same speed, but this one [red car] just stops earlier.
- "5 miles in 3 minutes would be one third of the trip."

Student Learning

Emily

- Saw that doubling both distance and time "worked"
- Did not have a way to show doubling with pictures
- Created a doubled journey as two smaller same-size journeys, with support
- Needed support to articulate how to justify same speeds

Lisa

- Saw that doubling both distance and time "worked"
- Showed doubling in her picture right away
- Created a doubled journey as two smaller same-size journeys
- Needed support to articulate how to justify same speeds

Student Learning



Lisa and Sara continued...

Self-assessment of Tiering: Were the number choices good ones?

Emily: Yes
Lisa and Sara: Yes, but...
Joanna: Yes

Implications

- This study shows how differentiating instruction supported student learning of reasoning with ratios across multiplicative concepts.
 - We argue that this is generally better for students than "one-size-fits-all" instruction.
- Differentiation is touted as a a component of inclusive mathematics classrooms in which equity is a priority (e.g., Boaler, 2019; Michael, 2015), but:
 - Differentiating is hard to do!
 - Need curricular materials
 - Need professional development

THANK YOU!

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- What IDR²eAM stands for: Investigating Differentiated Instruction and Relationships between Rational Number Knowledge and Algebraic Reasoning in Middle School
- http://www.indiana.edu/~idream/