## Students' Meanings for Extensive Quantitative Unknowns

From the $I D R^{2} e A M$ ProjecTeacher: Investigating Differentiated Instruction and Relationships between Rational Number Knowledge and Algebraic Reasoning in Middle School

## Single Unknown Problem Heather and Symone addressed in Episode 10

The height of Quint East Middle School is unknown. You have two measuring units, a toothpick length and a chopstick length. The toothpick length fits along the chopstick length 4 times. Imagine measuring the school's height in toothpicks, and then measuring it again in chopstick lengths. You would get two different values.
a) Draw a picture in JavaBars to show the two values for the height of the school. Your picture should show that the height is unknown, but indicate how the height could be measured with the two different units.
b) Pullout parts of your JavaBars pictures to identify the measurement units.
c) Describe how the two values for the height of the school in part (a) are related.
d) Let's let a letter stand in for the height of the school measured in chopstick lengths, and another letter stand for the height of the school measured in toothpick lengths. What equation can you write to show the relationship between these two unknowns? Explain what your equation means in terms of your picture.
e) Can you write a second equation with your unknowns from (d)? Explain what your equation means in terms of your picture.

## Transcript for clip of small group discussion, Episode 10

Teacher: So if it's the length of the chopstick, and T is the length of the toothpick, then you would have C, the length of the chopstick, times 4, is equal to the length of the toothpick. Is that true?
Heather: Yes.
Teacher: Show me in your picture. The length of the chopstick times ...
Symone: No.
Teacher: 4 is equal to the length of the toothpick.
Symone: No.
Teacher: No? How come, Symone?
Symone: Because toothpicks are way smaller than chopsticks.
Teacher: Yes, yes.
Heather: Yes, but you'd also get more at the end.
Teacher: Tell me what that means, you'd get more at the end?
Heather: Yes. Because if you add up how many toothpicks there are, you'd get more.
Teacher: Oh, get more in what?
Heather: You'd get more in the answer.
Teacher: Yes, what's the answer about?
Heather: How many chopsticks there are?
Teacher: How many chopsticks there are where?
Heather:: Altogether in the toothpicks.
Teacher: Yeah? And what's this problems about again? What quantity are we thinking about?

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Heather: The height of the school.
Teacher: Oh, ok. Does the height of the school have anything to do with how you're going to define these guys?
Heather: I don't know.
Teacher: Well, you just said something about there's going to be more toothpick lengths, right?
Heather: Yes.
Teacher: More toothpick lengths where?
Heather: In the height of the school.
Teacher: Oh, yes. Yes, and I think that's what this is representing. I think that's what this is representing to you, like there's more toothpick lengths than there are chopstick lengths.
Heather: Yes.
Teacher: In the height of the school? Ok, so that means T represents what?
Heather: The school.
Teacher: Something about the school. Something about toothpick lengths. How could we write it?
Heather: I don't know.
Teacher: Well, let's look at your picture. Where do you see T in your picture?
Heather: Right there [pointing to the 1-part bar labeled " 1 toothpick length"].

