The First Research Program: what it looks like in middle school

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First Research Program

- Students who
  - enter elementary school as perceptual counters;
  - interiorize one level of unit around 3rd grade;
  - enter middle school having interiorized one level of unit.
- About 30% of in-coming 6th grade students.

Plan

- Illustrate what it means to have interiorized one level of unit at 7th and 8th grade
- Demonstrate one of these student's experience in a CMP 8th grade unit
- Start a discussion around what is needed for students who have interiorized one level of unit

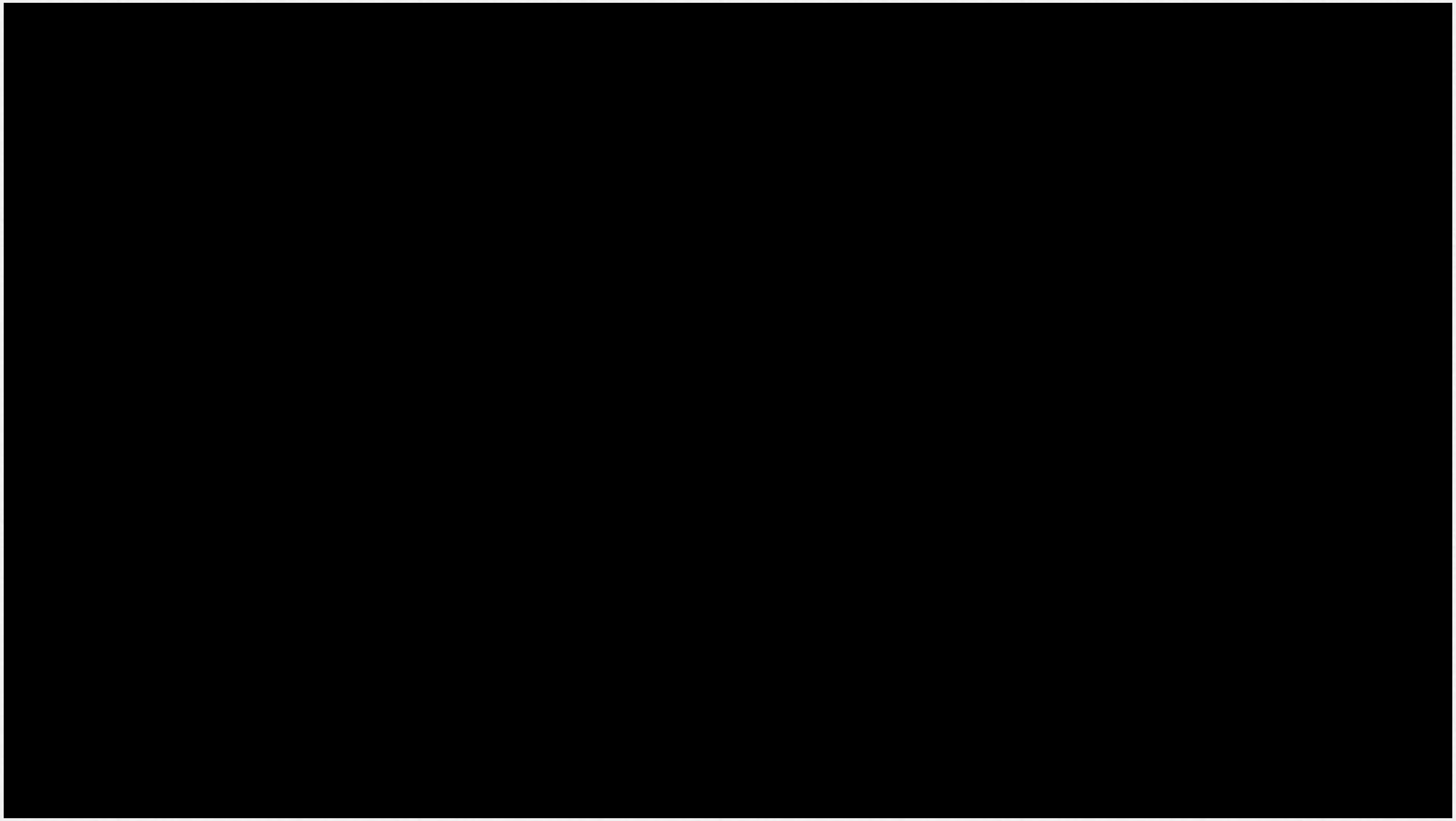
7th Grade Students Hal and Josiah

- Participated in first year of a 3-year project to study students' generalizing actions
- Selection interviews
- Goal to investigate the number of levels of units interiorized
- Hal had interiorized one level of unit
- Josiah had interiorized two levels of units

Candy Factory Task

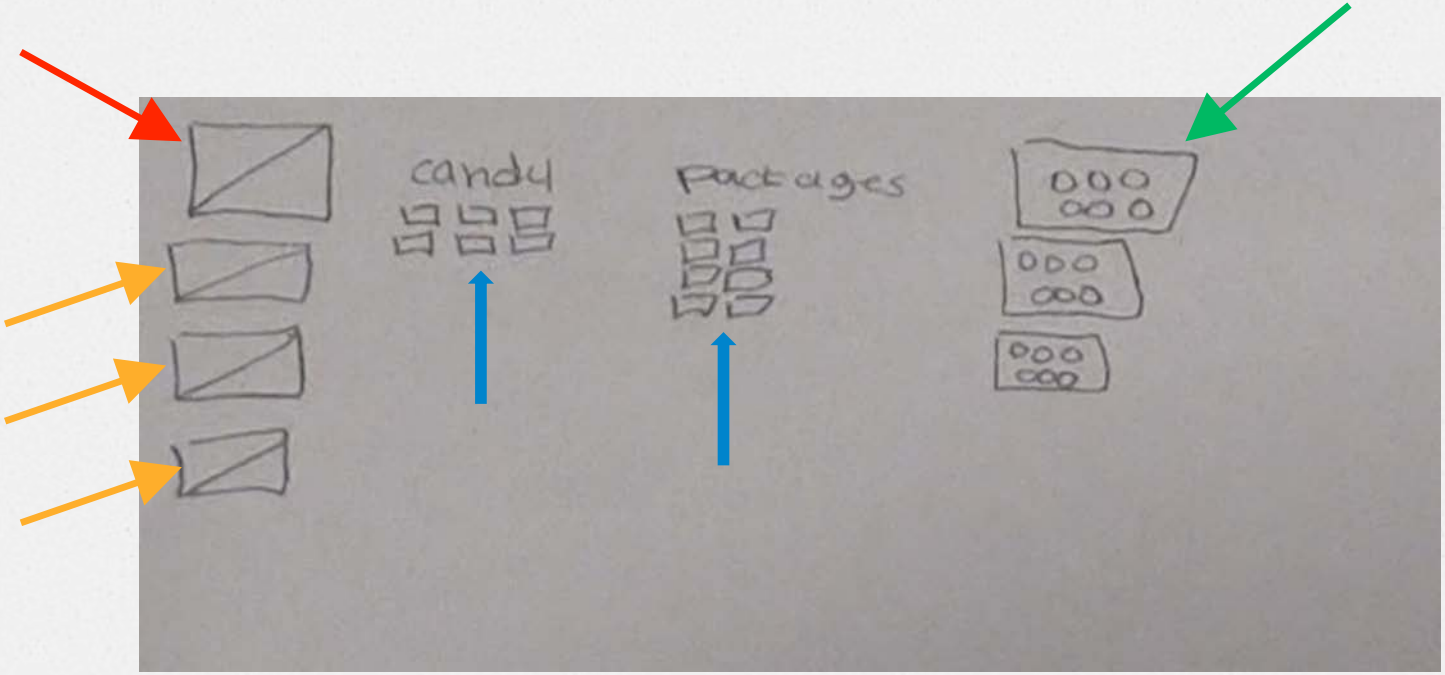
- A candy factory puts 6 candies in each package, puts 8 packages in each box, and puts 4 boxes in each crate. Make a picture of one crate.

6 candies go in a package, 8 packages in a box, 4 boxes in a crate



6 candies go in a package, 8 packages in a box, 4 boxes in a crate

Hal's Work



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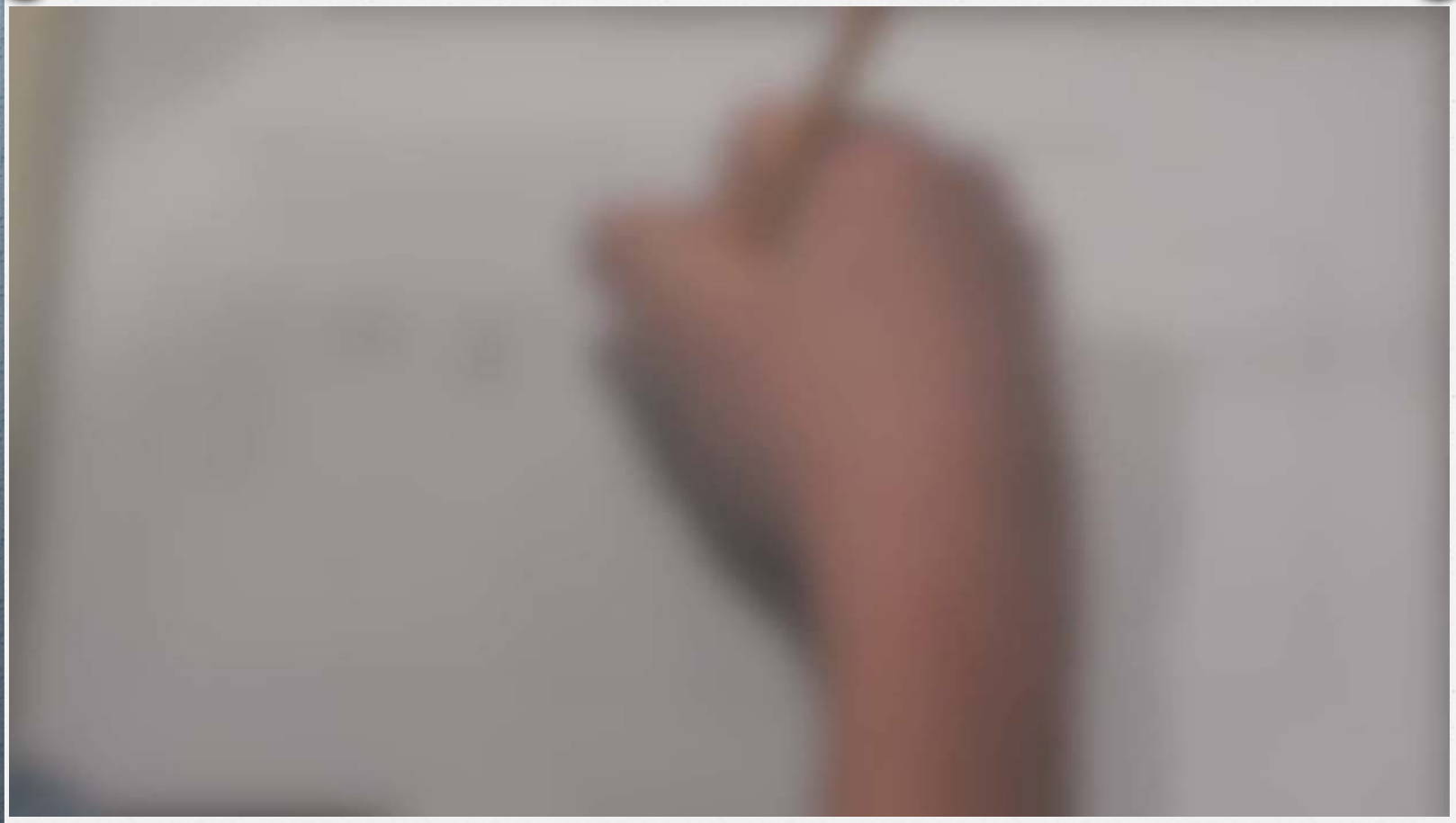

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8th grade student Alyssa

- Participant in a whole classroom design experiment co-taught with classroom teacher
  - 27-day classroom unit on equivalence using Connected Mathematics Project materials in a pre-algebra class
  - Goal: differentiate instruction for cognitively diverse learners, where diversity was primarily based on interiorization of units

Alyssa's regular, 8th grade pre-algebra class

Level of units	Numbers of students
1	5
2	13
3	2

Main mathematical activities before mid-unit interview

- Writing expressions based on quantitative situations (expressions for # of tiles around border of pools)
- Using expressions to interpret quantitative situations
- Determining whether expressions are equivalent
  - Substitution of values
  - Use of Distributive Property
- Adding expressions and writing equations with multiple expressions
- Substituting expressions in equations for profit

Mid-Unit Interview

- Isabel is participating in a swim-a-thon to raise money for her swim team. She collects money from her sponsors for every lap she swims. Each of her sponsors pledges to give her $10 to start, and another $2 for every lap she swims.
  a. Write an expression to represent how much money Isabel will earn for the swim team if she gets 15 sponsors.

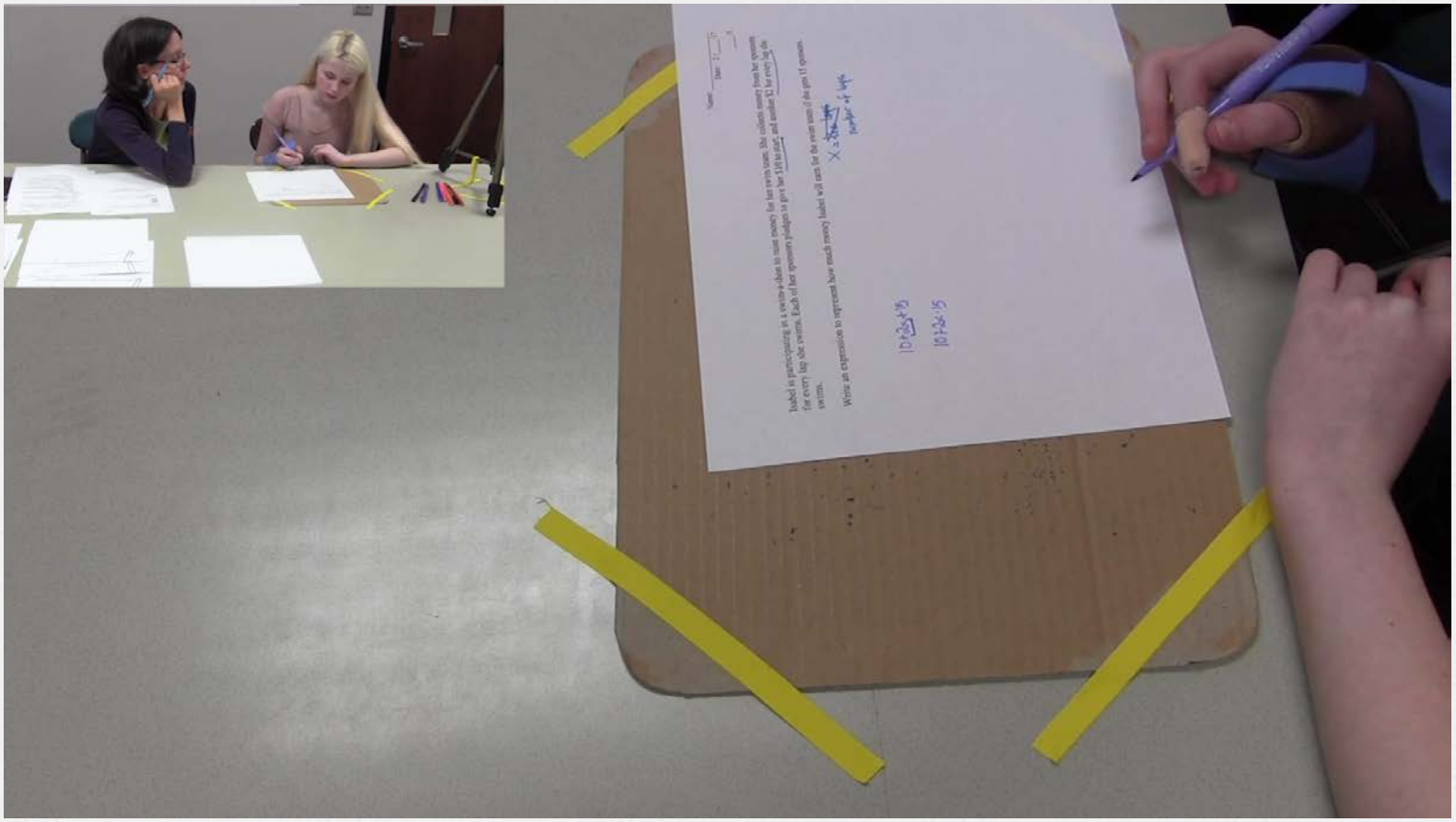
  b. Nikolas is also raising money. His sponsors pledge $4 for every lap he swims. Write an expression to represent how much money Nikolas will earn if he gets 15 sponsors.

Each of 15 sponsors pledges $10 to start and $2 per lap.

Alyssa's initial work

- "I would say you would do 10 and 2 but I don't know if that's right because you don't know how many laps for 15 sponsors."
- Identified x as "the laps" and then when questioned, "number of laps."
- Wrote:
  - 10 + 2x
  - 10 + 2x + 15 because "if she's getting more, then you're adding, is what I thought."
  - 10 + 2x*15, but that would be "way more money."

Each of 15 sponsors pledges $10 to start and $2 per lap.



Second-order observations about Alyssa

- She did not view 10 + 2x as the amount earned from one sponsor.
- Adding was a way to express getting more money.
- After she proposed 10 x 15, she conflated the amount from all 15 sponsors with an amount from 1 sponsor—she did not maintain the $150 as coming from 15 $10's.

Each sponsor pledges $10 to start and $2 per lap.

Moving away from unknowns

- 1 sponsor, 4 laps

Alyssa's activity	Alyssa's comments	My comments
$16 = 4 + 4 + 4 + 4	"I don't know why I did that."	May have iterated 4 laps 4 times instead of $2 4 times.
$12 = $10 + $2	"She has 10 to start with from 1 sponsor, and she's getting $2 per lap."	AH: How many laps would she swim if she got $10 plus $2? A [promptly]: One.

Each sponsor pledges $10 to start and $2 per lap.

Each sponsor pledges $10 to start and $2 per lap.

Moving to 2 sponsors

Alyssa's activity	Alyssa's comments	My interpretation
1 sponsor: $18 = $10 + $8	"I don't even remember what I did; I just got 8."	Coordinating 2 levels of units in activity to get $8.
2 sponsors: $28 = $20 + 8	"I still got 8 though. Because like if you do 2 four times and you plus it, you'll get 8."	Cannot double the $8 because it is produced in activity.

Curricular goals and Alyssa's mathematics

- A main goal of the curricular tasks was for students to develop embedded multiplicative relationships with unknowns, opening the way for the use of the Distributive Property.
  - E.g., 15(10 + 2x)
  - And: 15(10 + 2x) + 15(4x)
- Alyssa did not show evidence that this was in her immediate ways and means of operating. It is not clear to us that working on such tasks with students like Alyssa is effective at engendering this kind of reasoning.

Mathematics for Alyssa

- We need learning trajectories for students who have interiorized only a single level of unit on which we can base design of curricular materials.
- These materials need to respect students like Alyssa as mathematical thinkers in their own right.
- We see this as a critical aspect of equity.

Thank you! Shout-outs to:

- IDR^{2}eAM project members!
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- Les Steffe!
- PME-NA 2017 organizing committee!

Possible Questions for Discussion

1. Where is the field right now in serving students like Alyssa in middle school?
2. How might this research work in conjunction with other research aimed at greater equity in mathematics education?