Students' Meanings of Division with Fractions & Unknowns

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IDR²eAM Project: Investigating Differentiated Instruction and Relationships between Rational Number Knowledge and Algebraic Reasoning in Middle School

- **Study:**
  - how to differentiate instruction for cognitively diverse middle school students
  - how students' rational number knowledge and algebraic reasoning are related
- **Phase I (Yrs 1 & 2):** Conducted three 18-episode after school design experiments with 6-9 cognitively diverse middle school students

Purpose of talk

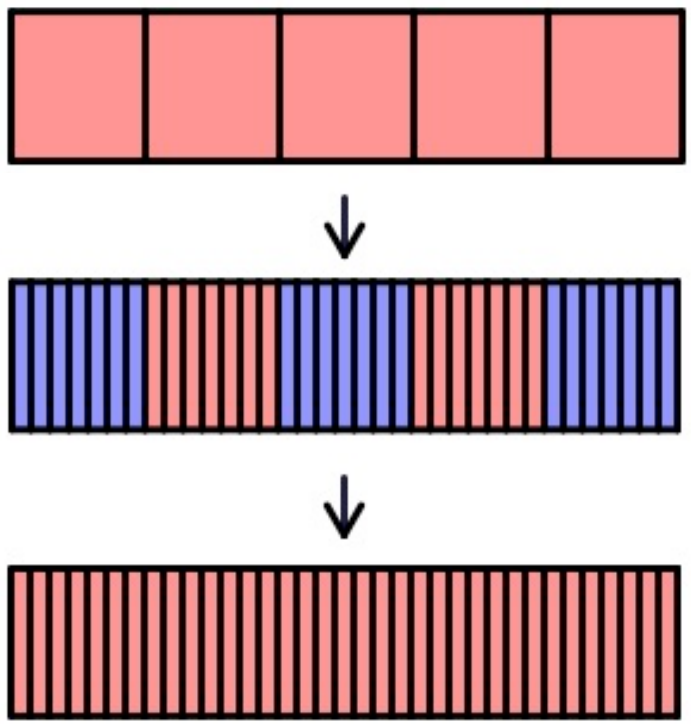
- We (members of the IDR^{2}eAM team) are currently exploring students' meanings of division with fractions and unknowns across the three experiments
- Communicate a piece of that today

Students' Multiplicative Concepts

- *Concept*: interiorized result of a scheme.
- *Composite unit*: a unit of units.
- *Units coordination*: distribute the elements of one composite unit across the elements of another composite unit.

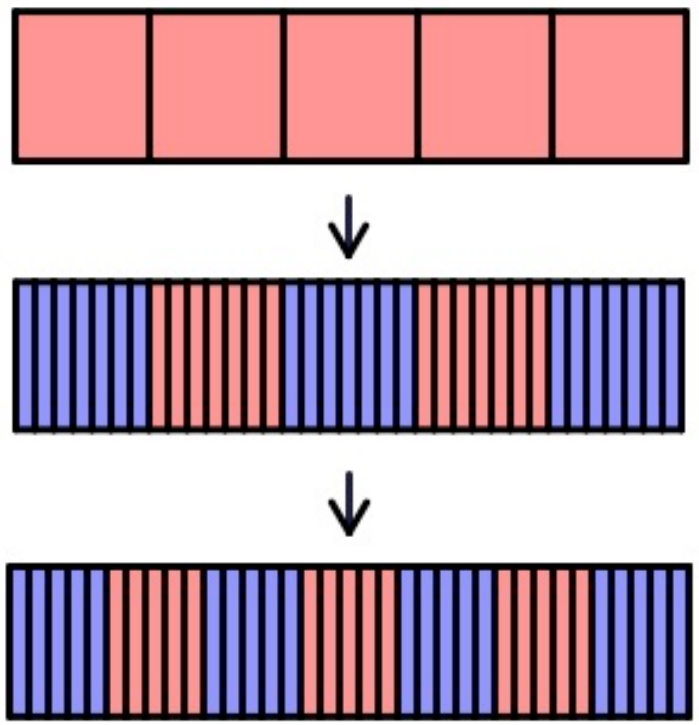
Students operating with the 2nd mult. concept (MC2 students)

- Can anticipate the coordination of two levels of units prior to activity
- Can produce three levels of units in activity



Students operating with the 3rd mult. concept (MC3 students)

- Can take three levels of units as given
- Can flexibly switch between three-levels-of-units structures



Algebraic Reasoning from a Quantitative Perspective

- Unknowns are potential measurements of quantities.

- Thinking of a quantitative unknown—say a length— requires being able to imagine a unit of units.

Students in the Experiments

	Experiment 1	Experiment 2	Experiment 3
MC 2	6	6 → 3	6 → 3
MC 3	3	3	3
Completers	9	7	6

Experiment 1, episode 11

- **Fern Sunflower Heights Problem.** The height of a sunflower is one-fourth the height of a fern. We don't know either height, so they are both unknowns.
  a. Draw a picture of the situation and describe what your picture represents.
  b. Write an equation for this situation that relates the two heights. Explain your equation in terms of your picture.
  c. Can you write another, different equation that relates the two heights? Explain this equation in terms of your picture.

Whole class discussion, episode 12

- q ÷ ¼ = c
  - q = fern ht (larger)
  - c = sunflower ht (smaller)
- $Connor_2$: **if you were to divide the [larger] height by ¼, it'd be like dividing it by 4, so then it'd equal the smaller height.**
- Teacher: So you're seeing dividing by 1/4 as sort of the same as dividing by 4?
- $Connor_2$: "Yeah!"

Tim_2 in experiment 1

- "I can't do division, I'm sorry I can't. I don't know how, I'm being dead serious; I don't know how."
- p ÷ m = 2 ½
  - p = rover's height
  - M = spot's height

Design Change

- **In Exp. 1 we found:**
  - Division by fractions and whole numbers undifferentiated for MC2 students
  - Division mysterious to MC2 students
  - Division by fractions procedural for MC3 students
- **Aim in Exp. 3:**
  - More explicit articulation of division relationships between quantitative knowns and between quantitative unknowns

Experiment 3, episode 8

- **Problem:** You have 6 feet of bubble gum tape and a single serving is ½ foot.
  a. How many people can get a single serving?
  b. Write a division sentence and tell what it means.
  c. What meaning of division are you using here?

Whole class discussion about what 6 ÷ 1/2 = 12 means

- Heather$_2$: 6 fits into 12 ½ times
- Yujeong$_3$: ½ fits into 6 12 times
- Emmett$_3$: 12 fits into 6 ½ times
- Class:
  - With teacher support, examines Heather's statement and changes it to 6 fits into 12 2 times.
  - Agrees on Yujeong's statement.

MC3 student Emmett: 12 fits into 6 ½ times

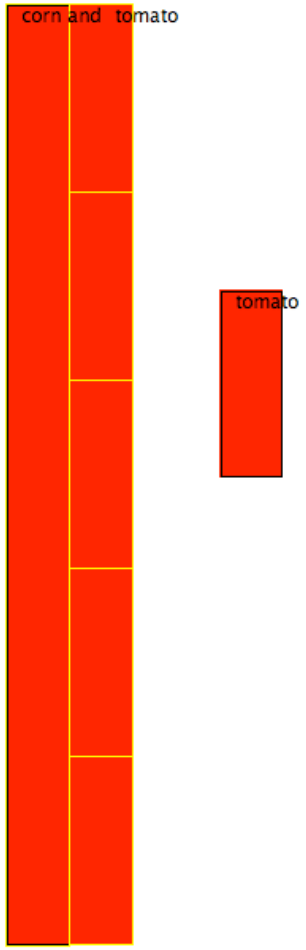
- Teacher: What division sentence would you write for your statement, Emmett?
  - $Emmett_3$: 12 • ½ = 6
  - $Heather_2$: 12 ÷ 6
  - $Emmett_3$: 12 ÷ ½ = 6
  - $Yujeong_3$: 12 ÷ 2 = 6
  - $Emmett_3$ with teacher support: 6 ÷ 12 = ½

Whole number relationships between unknowns

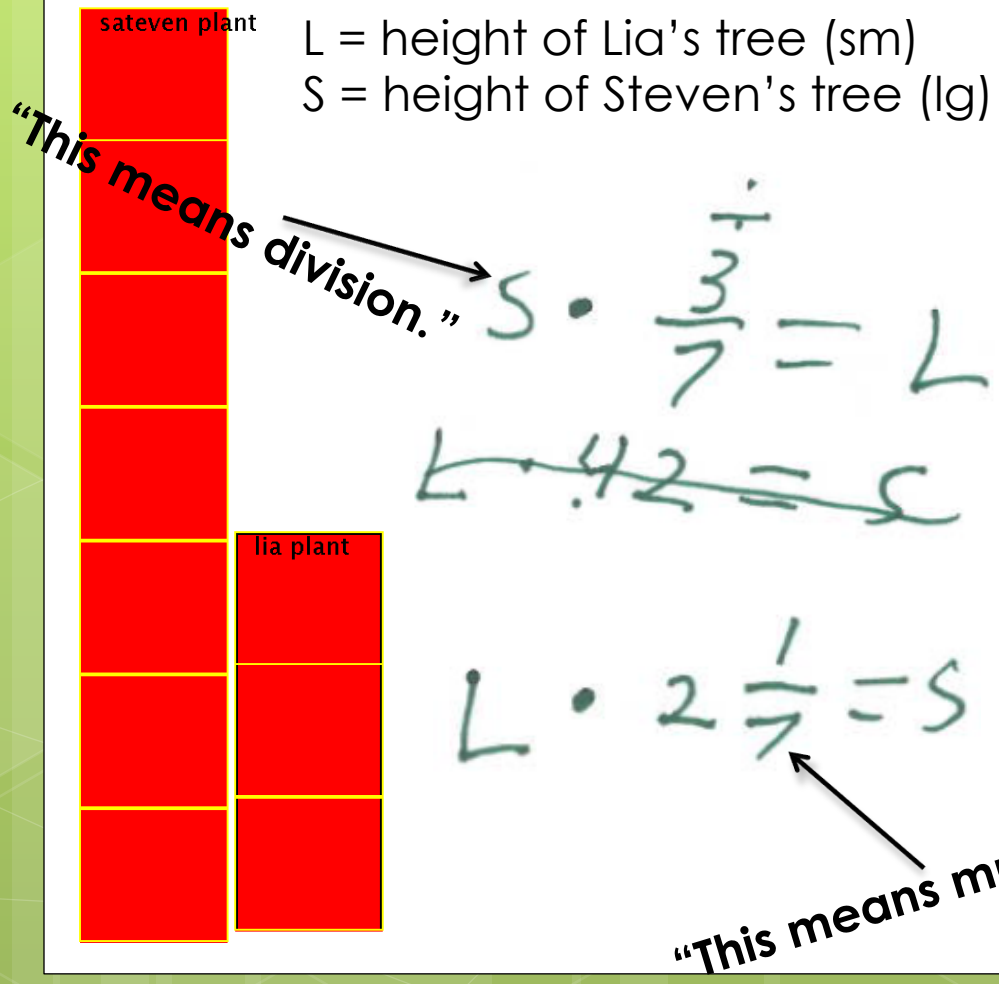
- **Corn Stalk Tomato Plant Heights Problem.** A tomato plant and corn stalk are growing in the garden, each of unknown height.

  The height of the corn stalk measured in inches is 5 times the height of the tomato plant measured in inches.

- q = height of corn stalk
- c = height of tomato plant
- 5c = q
  q ÷ c = 5
- q ÷ 5 = c



Milo$_2$'s Follow-up Interview



L = height of Lia's tree (sm)
S = height of Steven's tree (lg)

$S \cdot \frac{3}{7} = L$

$L \cdot .42 = S$

$L \cdot 2\frac{1}{7} = S$

- "Well, a fraction kind of means division. It's division but if you multiply it, it means division is the thing you multiply it by, and you're pulling out the part that you want."
- "It's the same thing but reversed. Instead of taking away you're adding."

Hmm…

- Are Milo's ideas about what multiplication and division are solely related to "multiplication makes bigger"? We think it's more complex than that.
  - He may associate partitioning with division and iterating with multiplication.
  - With proper fractional relationships, that association becomes muddled because taking 3/7 or 2 1/7 of a height requires both partitioning and iterating. So it's hard to classify what's what.
  - Furthermore, the fraction bar means division.

Parting thoughts…

- Although the ideas of measurement division with fractions and unknowns were a teaching tool recognizable by MC2 students, many of them did not spontaneously use the "fits in" idea with fractions and unknowns.
- But, being able to make sense of this idea and have conversations about it is one example of rich algebraic activity for MC2 students.

Thank you!

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- *What IDR²eAM stands for:* **I**nvestigating **D**ifferentiated Instruction and **R**elationships between **R**ational Number Knowl**e**dge and **A**lgebraic Reasoning in **M**iddle School
- **http://www.indiana.edu/~idream/**