Tiering Instruction on Ratios with Middle School Students

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IDR²eAM Project

Investigating Differentiated Instruction and Relationships between Rational Number Knowledge and Algebraic Reasoning in Middle School

Definition of DI: *proactively tailoring instruction to students' mathematical thinking while developing a cohesive classroom community* (cf. Tomlinson, 2005)

Teaching Practices for DI

- (1) using research-based knowledge of students' mathematical thinking
- (2) providing purposeful choices and different pathways
- (3) inquiring responsively during group work
- (4) attending to small group functioning
- (5) conducting whole classroom discussions across different thinkers

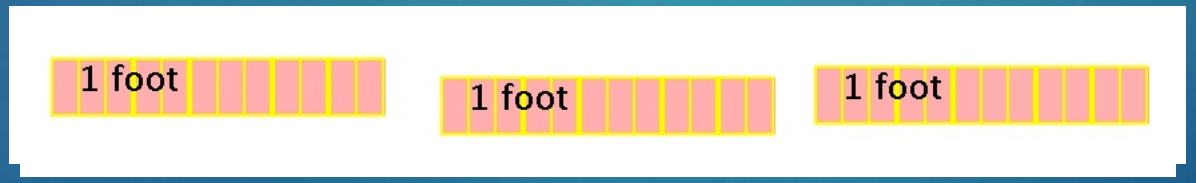
- (1) and (3) are about the heart of differentiating: GETTING TO KNOW STUDENTS' THINKING

From Hackenberg, Creager, and Eker (under review)

Students' Multiplicative Concepts

- Students' multiplicative concepts are based on how they coordinate units and maintain this coordination in mathematical activity.

- Unit: discrete "one," length, measurement unit
- Composite unit: a unit of units.
- Units coordination: distribute the units of one composite unit across the units of another composite unit.



Three Multiplicative Concepts: MC1, MC2, MC3

- Influence
  - fractions knowledge (e.g., Steffe & Olive, 2010)
  - reasoning with signed numbers (e.g., Ulrich, 2012)
  - equation writing (e.g., Hackenberg & Lee, 2015),
  - combinatorial reasoning (e.g., Tillema, 2014)
- Transition between concepts can take 2 years (Steffe & Cobb, 1998; Steffe & Olive, 2010)

Three Multiplicative Concepts

MC	Students' unit structures	Students' reasoning on the Crate Task: Four cans of juice are in a package; 8 packages are in a box; 6 boxes are in a crate. How many cans in the crate?	Approx. % entering 6th grade (Steffe, 2017)
1	• Can take one level of units as given • Create units coordinations in activity	• Need to build up from ones to create and nest quantities • Does not keep multiple levels in mind when operating further—e.g., might iterate 4 eight times but not recognize result as a box	30%
2	• Can take two levels of units as given • May coordinate three levels of units in activity	• Can keep track of eight 4s as a box, but box becomes a unit of 32 1s. • Often conflate boxes and packages when working with a crate	30%
3	• Can take three levels of units as given • Can flexibly switch structures.	• Can usually move flexibly among packages, boxes, and crate without conflation • Can see a crate as 6 boxes of 32 and as 48 packages of 4 cans	40%

Classroom Unit Overview

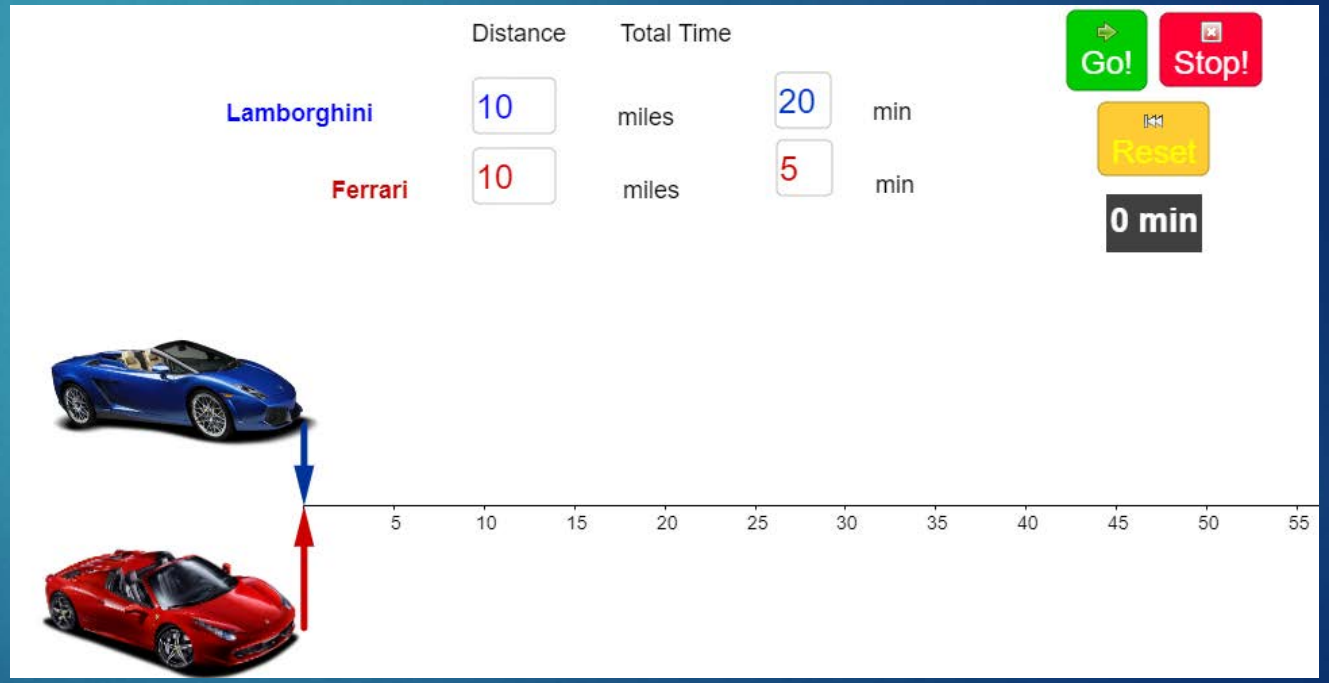
- Comparing and Scaling unit from the Connected Mathematics Project (CMP 3):
  - Ratios and proportional reasoning
  - 18 regular 7th grade mathematics students (plus 2 not participating)
  - 5 MC1, 9 MC2, 4 MC3
  - 26 days
  - 3 investigations

Inv.	Topic	Days	Differentiation strategies used
1	Quantifying orangeyness	1-8	Individual students pulled out from heterogeneous groups (jigsaw), tiering instruction, norms development
2	Quantifying speed	9-18	Tiering instruction, whole class discussion across different thinkers/problems
3	Understanding percentages	19-26	Choice of project topics, tiering within topic

Tiering Instruction

- DEFINITION: designing different problems (or sequences of problems) for different groups of students based on conjectures about what will support students' learning needs (Tomlinson, 2005).

Usually follows providing students with choices or getting to know student thinking in some way, such as through responsive inquiry.



https://www.geogebra.org/m/J434Kb54

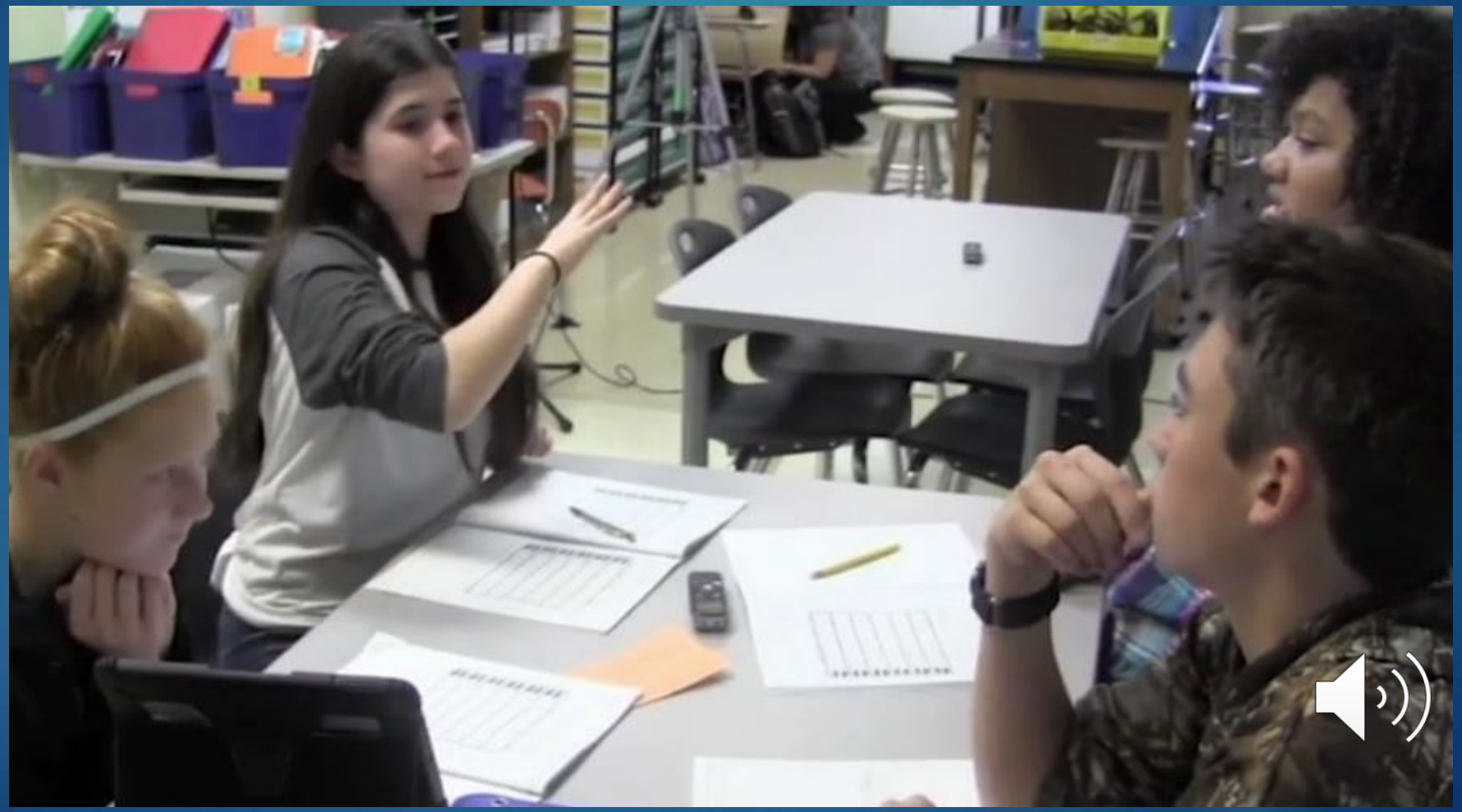
Making cars go the same speed

- Days 12 & 13
- Blue car goes ___ miles in ___ minutes. Make the red car go the same speed. Draw a picture to explain/justify

Tiering plan

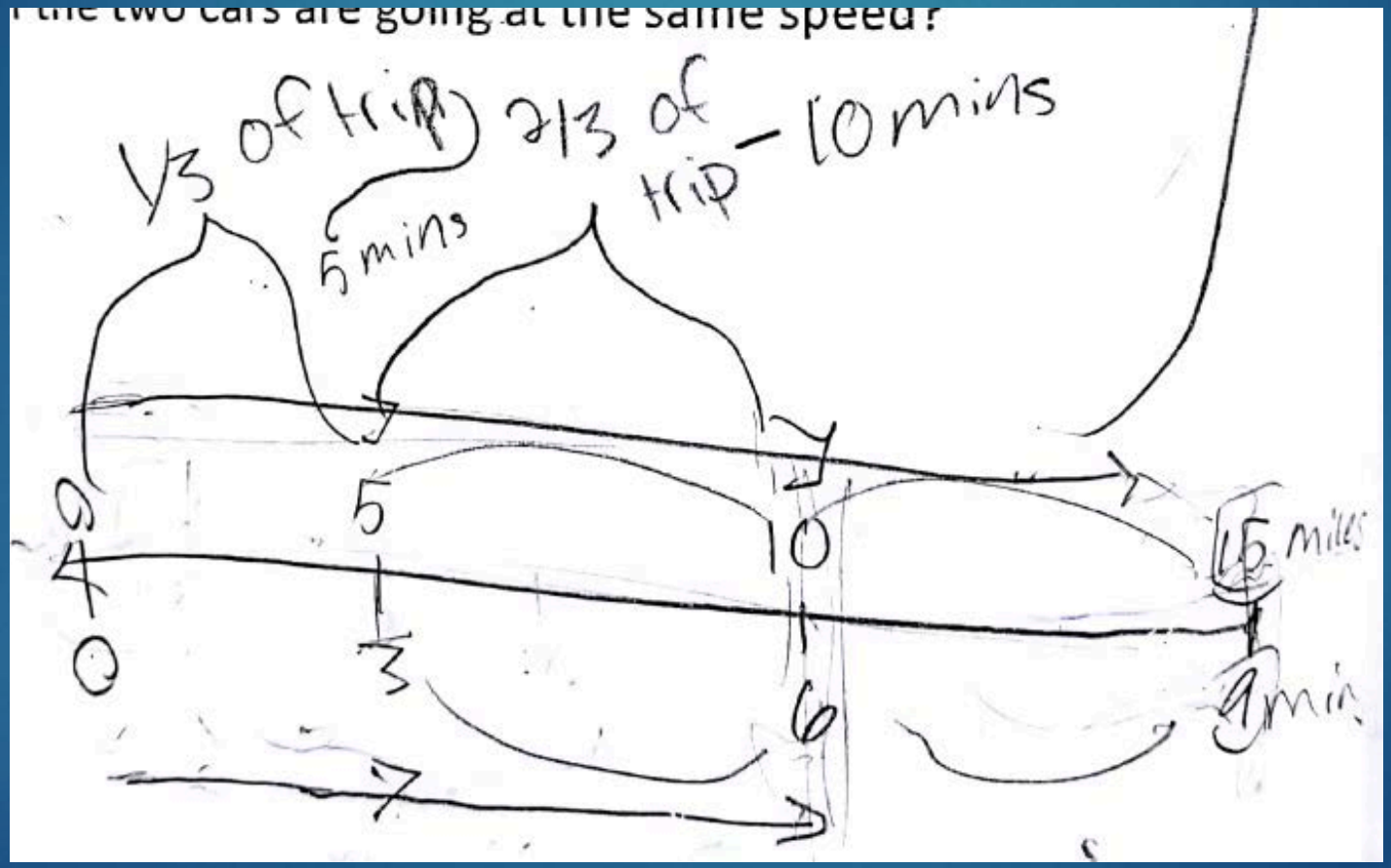
MC	Numbers	Our Rationale
1	18 miles in 3 minutes	Whole number unit ratio (6 miles in 1 min)
2 Lisa, Sara	15 miles in 6 minutes	Mixed number unit ratio with ½ (2.5 miles in 1 min)
3 Joanna	15 miles in 9 minutes	Unit ratio hard to work with as a decimal (5/3 miles in 1 min)

Joanna's Group: 15 miles in 9 min



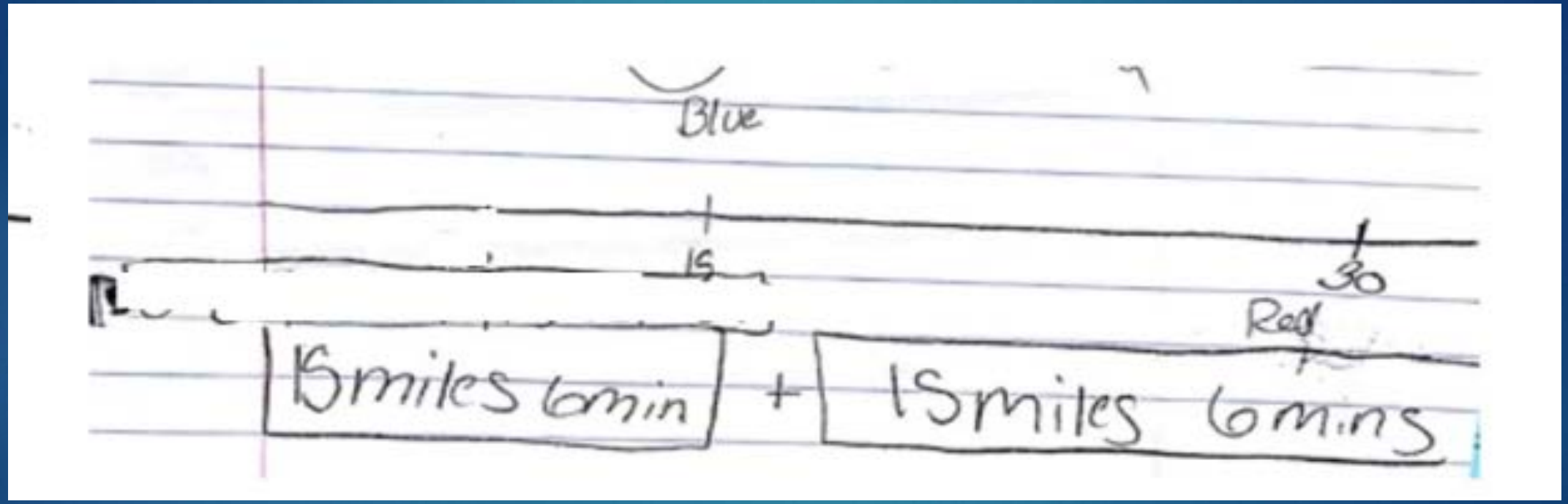
Joanna: “Any numbers where the miles would reduce to five and the minutes would reduce to three, because it’d be the same ratio to each other.”

Joanna's Picture: 15 miles in 9 min

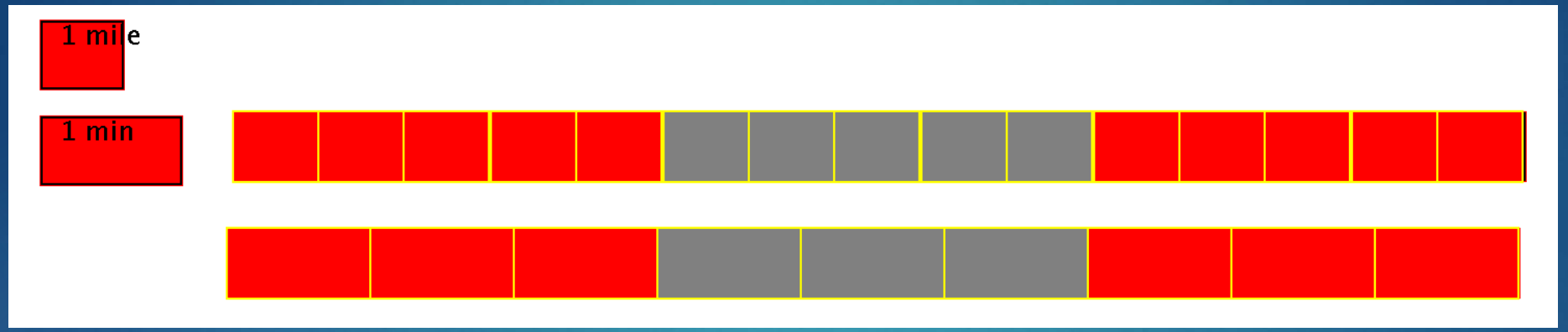


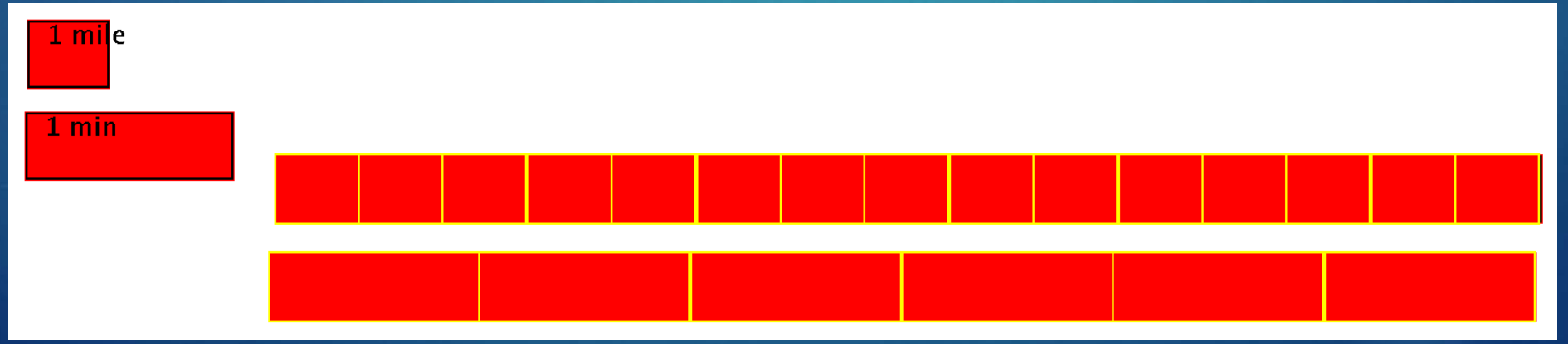
Sara and Lisa: 15 miles in 6 min

Lisa's Picture: 15 miles in 6 min

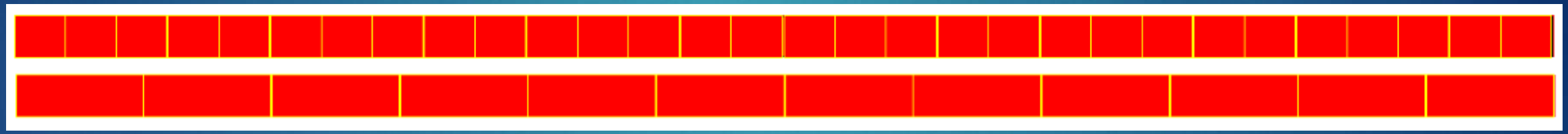


Relationship of Student Work to Multiplicative Concepts





Sara and Lisa, continued



How Tiering affected students

- Homogeneous grouping:
  - Each student could work at their edge, struggling together
  - Teachers could tailor problems to the needs of the students
- Commonalities:
  - No one had it all worked out
  - Each needed time to reason why their answers gave the same speed
- Were the numbers choices good for them?
  - Joanna: Yes.
  - Sara and Lisa: Yes, but.

Thank you!

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- *What IDR²eAM stands for:*
  Investigating Differentiated Instruction and Relationships between Rational Number Knowledge and Algebraic Reasoning in Middle School
- http://www.indiana.edu/~idream/

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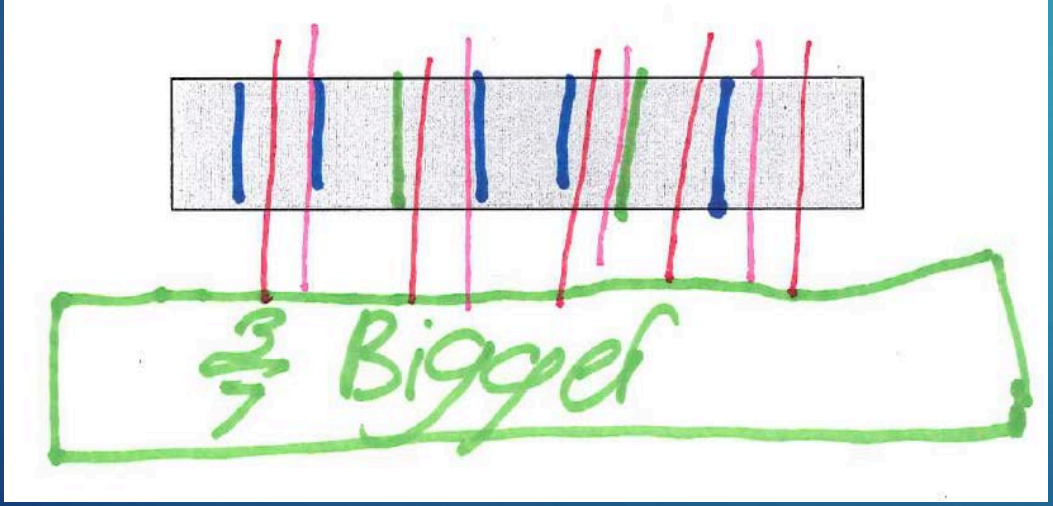
QUESTIONS for Discussion

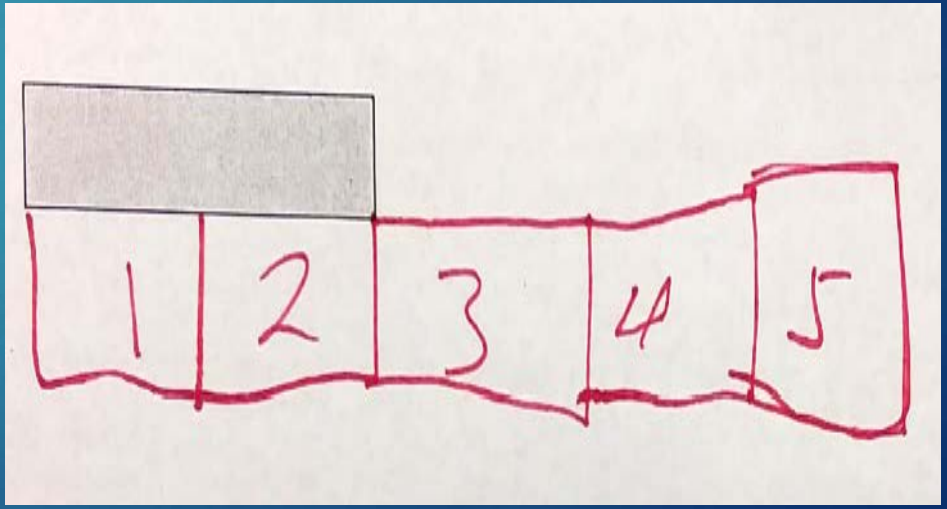
- How are Universal Design for Learning and Differentiating Instruction related?
- What different needs do students identified with LDs require that others don't?
- What are the implications of these research programs for teaching and classroom practice?
  - How do we determine most important learning needs for students?
  - How do teachers/researchers develop an organized framework of student thinking to work from?
  - How do teachers/researchers balance using a developmental framework with seeing variety within students a particular point in the framework?
  - How do we progress monitor student learning?

Changes in Lisa's Fractions Knowledge

From Initial Interview: "Here's a candy bar. There's another bar that's 3/7 of that bar. Can you draw the other bar."



From Follow-up Interview: "This bar is 2/5 of another bar. Can you draw the other bar?"



Students' multiplicative concepts in classroom experiments

MC	8th (participating)	8th (comparison)	7th (participating)	7th (comparison)
1	5	3	5	3 (6)
2	13	15	9	8
3	2	5	4	6
Totals	20	23	18	17 (20)

Classes: 8th grade regular pre-algebra; 7th grade regular math