Tiers, not Tears! One Strategy for Differentiating in Middle School

From the IDR²eAM Project: **I**nvestigating **D**ifferentiated Instruction and **R**elationships between **R**ational Number Knowl**e**dge and **A**lgebraic Reasoning in **M**iddle School

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7TH GRADE MATHEMATICS CLASS

SAME SPEED TASK: The blue car travels _____ miles in _____ minutes. Make the red car travel at the same speed as the blue car, but the red car will travel a different amount of miles and a different amount of minutes. [Later we asked them to justify their results with a picture and explanation.]

Fractions Knowledge	Orangeyness Task	Numbers
• Fractions are parts within wholes or parts out of wholes—no length meaning	• Not fluidly iterating two quantities as a composed unit.	• 18 miles in 3 min • Whole number unit ratio (6 miles in 1 minute)
• Beginning to think of fractions as lengths • Improper fractions are not numbers	• Iterating two quantities as a composed unit.	• 15 miles in 6 min • Mixed number unit ratio with ½ (2.5 miles in 1 minute)
• Think of fractions as lengths • Improper fractions are numbers	• Iterating two quantities as a composed unit • Making unit ratios	• 15 miles in 9 min • Unit ratio hard to work with as a decimal (5/3 miles in 1 minute)

Lisa and Sara's group (transcript), Day 11: Their blue car travels 15 miles in 6 minutes.

Sara: It's impossible.
Lisa: Ms. Hackenberg, It's impossible. We give up.
AH: I'm coming back, just a second. [She talks with another group.]
Lisa [when Ms. H returns]: It's impossible, even when we do that, 15.1 and 6.1, because it's not 15 or 6, so.
AH: Okay, so you really think it's impossible?
Lisa: Yeah.
Sara: Yeah, unless you do, like...
AH: So, two cars can't go the same exact speed but go different distances and times?
Sara: They probably could, but I can't figure it out.
Lisa: When you say we can't use 15 or 6, it's kind of hard.
AH: Right. [pause] All right, well that's good to know that it's hard.
Sara: Unless you double it, and it's going the same speed, you just doubled it.
AH: Wait, what do you mean, Sara?
Lisa: 30 and 12?
AH: Try that. What does that give you?
Sara: 30 and 12?

Lisa: Yeah, it's doubling.
AH: Double the distance and double the time. Do you think you'll go the same speed or no?
S: I don't know. [She watches as Lisa tries these numbers in the app.] I mean they're not going the same exact speed but they're going the same speed, just…
AH: Do you think it'll be the same speed, not the same exact--?
Sara [standing up]: I figured the system out!
Lisa: Oh wow, it worked! Okay.
AH: [laughing]: Write that down.
Sara: I'm smart. What do I get?
Lisa: So, do we just put tie [do they write "tie" in their tables]?
Sara: Oh I did it!

8TH GRADE PRE-ALGEBRA CLASS

Differentiated Versions of Investigation 4.1 from Connected Math[1]

All students did the following problem:

1) Pauline the pool worker comes in to fill a pool at noon on Monday. Her hose fills the pool at 12 gallons per minute.
   a. What equation could you write to represent the amount of water in the pool, w, after t minutes since noon?
   b. How many gallons will be in the pool after 30 minutes? An hour? 90 minutes?
   c. If the pool holds 5400 gallons, how much time does it take to fill the pool?
   d. Describe the pattern of change in the amount of water in the pool and the amount of minutes that have passed.
   e. What does the graph for this situation look like? Can you determine what the graph will look like without plotting points?

Then each was assigned one of the following:

Version A	Version B
Remember Pauline the pool worker? She is still working at her job maintaining pools. 2) When Pauline comes in at noon on Tuesday to fill another identical pool, fortunately she finds that there are already 1080 gallons of water in the pool. She will still fill the pool at a rate of 12 gallons per minute. This equation expresses the amount of water, w, in the pool after t minutes since noon: $w = 12t + 1080$ a. What does the 12 mean in the equation?	Remember Pauline the pool worker? She is still working at her job maintaining pools. 3) When Pauline comes in at noon on Wednesday to fill another identical pool, the pool is empty. She gets a phone call and does not start filling the pool for 5 minutes. This equation expresses the amount of water, w, in the pool after t minutes since noon: $w = 12(t - 5)$ a. What does the 12 mean in the equation? b. Why is 5 subtracted from t in this equation? c. Write an expression for w that is equivalent to the original expression in the equation above.

[1] Lappan, G., Phillips, E. D., Fey, J. T., Friel, S. N., Grant, Y., & Stewart, J. (2014). *Connected mathematics 3*. Boston, MA: Pearson.

b. Why is 1080 added to 12*t* in the equation?
c. How many gallons will be in the pool after 30 min? After an hour?
d. If the pool holds 5400 gallons, how much time does it take to fill the pool?
e. Describe the pattern of change in the amount of water in pool and the amount of minutes that have passed.
f. What does the graph for this situation look like? Can you determine what the graph will look like without plotting points?
g. An engineer comes in partway through the day and figures out how many gallons are in the pool. She does it again 15 minutes later. How much will the amount of water change in that 15 minutes? Does it matter when during the day she arrives?

What information does this new expression tell you about the situation?
d. How many gallons will be in the pool after 30 min since noon? After an hour since noon?
e. If the pool holds 5400 gallons, how much time does it take to fill the pool?
f. Describe the pattern of change in the amount of water in pool and the amount of minutes that have passed.
g. What does the graph for this situation look like? Can you determine what the graph will look like without plotting points?
h. An engineer comes in partway through the day and figures out how many gallons are in the pool. She does it again 15 minutes later. How much will the amount of water change in that 15 minutes? Does it matter when during the day she arrives?
i. What if the engineer measures the amount of water in the pool, and then measures it again ½ of a minute later. How much will the amount of water change in that ½ minute? Explain.

4) When Pauline comes in at noon, a pool identical to the one on the other days is full. She is supposed to empty the pool, and it empties at 12 gallons per minute. This equation expresses the amount of water, *w*, in the pool after *t* minutes since noon:

$$-12t = w$$

a. Explain why the equation involves -12. What does that mean?
b. When t is 30, w is -12(30)=-360. What do the values of t = 30, w = -360 mean in the situation?
c. The pool holds 5400 gallons. Find out how much water is left in the pool 2 hours after noon.
d. Describe the pattern of change in the amount of water in the pool and the amount of minutes that have passed.
e. What does the graph for this situation look like? Can you determine what the graph will look like without plotting points?
f. An engineer measures the amount of water in the pool, and then measures it again ½ minute later. How much will

5) When Pauline comes in at noon on Friday, a pool identical to the one on the other days is full. She is supposed to empty the pool, and it empties at 12 gallons per minute. However, first she has to change the filter for the pool, which takes 10 minutes. This equation expresses the amount of water, *w*, in the pool after *t* minutes since noon:

$$w = -12(t - 10)$$

a. Explain why the equation involves -12. What does that mean?
b. Why is 10 subtracted from t in this equation?
c. Write an expression for w that is equivalent to the original expression in the equation above. What information does this new expression tell you about the situation?
d. When t is 30, w is -12(30-10)=240. What do the values t=30, w=240 mean in the situation?
e. The pool holds 5400 gallons. Find out how much water is left in the pool 2 hours after noon.
f. Describe the pattern of change in the amount of water in the pool and the amount of minutes that have passed.
g. What does the graph for this situation look like? Can you determine what the graph will look like without plotting points?

the amount of water change in that ½ minute? Does it matter which ½ minute she measures during the day?	h. What is the y-intercept for this graph? What could the y-intercept mean in this situation?

Kathy's work on Problem #3 (Transcript), Day 22: $w = 12(t - 5)$

RJ: So what does that point mean [where Kathy's graph crosses horizontal axis]?
Angela: Five [minutes after noon].
Kathy: No, six, so that would be 12:06 because she wasted 5 minutes.
RJ: Right where that hits right there, what is w [number of gallons of water in pool]?
Kathy: Zero…twelve, it's twelve.
RJ: Right down here, right at this point [points to Angela's paper, where $t = 5$, $w = 0$]?
Angela: That's zero.
Kathy: No, at six it would be twelve.
RJ: At six it would be twelve, but you're saying at this point where it intersects, it would be zero?
Kathy: I'm confused.
RJ: If I just plotted this point right here—
Kathy: Oh wait, no, that's not correct. It would have to be up higher…