WHILE YOU WAIT, TRY OUT THE RACES APP!

Try It out!
- Make the red car go slower when both cars travel the same distance.
- Make the red car go slower when both cars travel for the same amount of time.
- Make the cars travel the same speed when they travel different distances/times.

[Diagram showing two cars with distances and times: Lamborghini (10 miles, 20 min) and Ferrari (10 miles, 5 min).]

www.geogebra.org: Search Janet Bowers NewRace
Direct link: https://www.geogebra.org/m/vabtrttr
TIERS, NOT TEARS!
ONE STRATEGY FOR DIFFERENTIATING IN MIDDLE SCHOOL
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OUR GOAL TODAY

- Share how we **tiered instruction** when working on ratio reasoning and linear functions with middle school students, and discuss challenges and strategies for using it in your own classrooms.

**DEFINITION of DIFFERENTIATING INSTRUCTION**: proactively tailoring instruction to students’ mathematical thinking while developing a cohesive classroom community (adapted from Tomlinson, 2005)
### SOME TEACHING PRACTICES FOR DIFFERENTIATING

<table>
<thead>
<tr>
<th>Providing Purposeful Choices</th>
<th>Designing Different Pathways</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Choice Problems</strong> (Land, 2017)</td>
<td><strong>Tiering Instruction</strong> –</td>
</tr>
<tr>
<td>- same problem with number choices</td>
<td>- same big idea</td>
</tr>
<tr>
<td>- students choose numbers that are a good level of challenge for them</td>
<td>- different sequences of problems</td>
</tr>
<tr>
<td><strong>Parallel Tasks</strong> (Small &amp; Lin, 2010)</td>
<td>- teacher designs/chooses based on formative assessment</td>
</tr>
<tr>
<td>- same big idea, 2-3 problems</td>
<td><strong>Inquiring Responsively in Groups</strong></td>
</tr>
<tr>
<td>- students choose problem</td>
<td>- listen, observe, ask questions</td>
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<tr>
<td></td>
<td>- try to understand how students are understanding problems</td>
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<tr>
<td></td>
<td>- pose questions, adaptations of problems, follow-up problems</td>
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</tbody>
</table>
DEFINITION: designing different problems (or sequences of problems) for different groups of students based on conjectures about what will support students’ learning needs (Tomlinson, 2005).

Usually follows providing students with choices or getting to know student thinking in some way, such as through responsive inquiry.
TWO CLASSROOMS

Spring 2017
- 8th grade pre-algebra
- 21 students
- 27 days

Fall 2017
- 7th grade mathematics
- 18 students
- 26 days
Try It out!

- Make the red car go slower when both cars travel the same distance.
- Make the red car go slower when both cars travel for the same amount of time.
- Make the cars travel the same speed when they travel different distances/times.
MAKING THE RED CAR GO SLOWER

- Source: Joanne Lobato’s Math Talk project, www.mathtalk.sdsu.edu

- Days 9-11 (out of 27 days)
- How do you measure fastness? How do you tell one car is faster?
- Make the red car go slower if:
  - Both travel the same distance
  - Both travel for the same time.
  - Write a general rule for how to make this happen.

- Making the red car go slower when distance values were the same was not problematic.
- Making the red car go slower when time values were the same was harder for some students.
Days 12 & 13 (out of 27 days)

Blue car goes ___ miles in ___ minutes. Make the red car go the same speed. Draw a picture to explain/justify.

## Tiering plan

<table>
<thead>
<tr>
<th>Fractions Knowledge</th>
<th>Orangeyness Task</th>
<th>Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>*Fractions are parts within wholes or parts out of wholes—no length meaning</td>
<td>*Not fluidly iterating two quantities as a composed unit.</td>
<td>18 miles in 3 min</td>
</tr>
<tr>
<td></td>
<td></td>
<td>*Whole number unit ratio</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(6 miles in 1 minute)</td>
</tr>
<tr>
<td>*Beginning to think of fractions as lengths</td>
<td>*Iterating two quantities as a composed unit.</td>
<td>15 miles in 6 min</td>
</tr>
<tr>
<td>*Improper fractions are not numbers</td>
<td></td>
<td>*Mixed number unit ratio with $\frac{1}{2}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(2.5 miles in 1 minute)</td>
</tr>
<tr>
<td>*Think of fractions as lengths</td>
<td>*Iterating two quantities as a composed unit</td>
<td>15 miles in 9 min</td>
</tr>
<tr>
<td>*Improper fractions are numbers</td>
<td>*Making unit ratios</td>
<td>*Unit ratio hard to work with as a decimal ($5/3$ miles in 1 minute)</td>
</tr>
</tbody>
</table>
MAKING CARS GO THE SAME SPEED: EMILY
18 MILES IN 3 MINUTES
EMILY’S PICTURES
18 MILES IN 3 MINUTES
MAKING CARS GO THE SAME SPEED: SARA & LISA
15 MILES IN 6 MIN
MAKING CARS GO THE SAME SPEED: SARA & LISA
15 MILES IN 6 MIN
Lisa's Pictures
15 Miles in 6 Min

75

3.75 miles in 15 min + 3.75 miles in 15 min

5 miles 6 min + 15 miles 6 mins
MAKING CARS GO THE SAME SPEED: JOANNA  
15 MILES IN 9 MIN
JOANNA’S PICTURE
15 MILES IN 9 MIN

[Diagram showing distances and times]

[Image of students discussing a math problem]
ALL groups solved their problem. **What were their ways of thinking and how were they different?**

- **Emily**
  - Doubling would work, but she did not know how to show that with pictures.
  - Created a way to show a doubled journey as two smaller same-size journeys.
  - Did not create other solutions.

- **Sara and Lisa:**
  - Doubling (and then tripling, quadrupling) was a process that became general for them on Day 12.
  - But they did not create smaller distance-time pairs until prompted, and then they only halved.
  - No thirding observed from them until questioned further.

- **Joanna:**
  - Multiples of the least common factors of the given values would produce the same speed.
  - She knew this was general and could produce many same speed pairs.
  - She never considered doubling, but that is included in her general solution.
**COMPARING THE GROUPS**

- *Were the numbers choices good for them? Yes, but...*
  - Emily: Yes.
  - Sara and Lisa: What about 14 miles in 4 min?
  - Joanna: Yes.
- *What happened in class discussion? All presented!*
Section 4.1 of Say it With Symbols introduces the following situation:

Magnolia Middle School needs to empty their pool for resealing. Ms Theodora’s math class decides to collect data on the amount of water in the pool and the time it takes to empty it.

The class writes the following equation to represent the amount of water \( w \) (in gallons) in the pool after \( t \) hours.

\[
w = -250(t - 5)
\]

- What information does the -250 represent?
- What units should you use for -250?
- What information does \((t - 5)\) represent? What units should you use for \((t - 5)\)?
- What units should you use for \(-250(t - 5)\)? Explain.
On your handout you have five questions written as alternatives to the previous problem.

The first one was given to everyone, and everyone solved it.

On the left side are questions we consider accessible to students who struggled to nest units within units. On the right side are questions we gave to students who had more success nesting units.
Darrin worked on the questions in the left column. In interviews and school work, he struggled to coordinate multiple quantities but did well connecting context to representations and could simplify situations to help himself be successful.

- The big idea: we wanted students to think about the coordination of both quantities (time, water in pool) as they were changing.
  - NOTE: all students in this class could graph equations by creating tables of points. We wanted to push them to think more about relationships between changing quantities.
initial graph of $w = 12t + 1080$
Kathy worked on the questions from the right column. Kathy didn’t see herself as good at math but was very engaged in class, thought deeply and asked great questions.
KATHY

Kathy’s initial graph for \( w = 12(t - 5) \)

Second attempt, ‘starting’ at 12 gallons, 6 minutes
What’s a topic you will be working on soon with your students?
How could tiering fit with this topic?

What resources do you use for finding tasks?
- We used CMP and Joanne Lobato’s project, mathtalk.sdsu.edu.
THANK YOU!

- With BIG thanks to all other members of the IDR²eAM project team, past and present: Mark Creager, Anna Dinndorf, Ayfer Eker, Sharon Hoffman, Robin Jones, Rob Matyska, Musa Sadak, Serife Sevinc, Pai Suksak, Ryan Timmons, Erol Uzan

- With BIG thanks to Patti Walsh and Marie Johannisson

- What IDR²eAM stands for:
  Investigating Differentiated Instruction and Relationships between Rational Number Knowledge and Algebraic Reasoning in Middle School

- http://www.indiana.edu/~idream/
RESOURCES

- Races app: [https://www.geogebra.org/m/J434Kb54](https://www.geogebra.org/m/J434Kb54)
- Carol Tomlinson’s Website: [http://www.caroltomlinson.com/](http://www.caroltomlinson.com/)
- Joanne Lobato’s Math Talk project: [https://mathtalk.sdsu.edu/](https://mathtalk.sdsu.edu/)
- Our IDReAM website: [http://www.indiana.edu/~idream](http://www.indiana.edu/~idream)