Students' Meanings for Extensive Quantitative Unknowns

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Rationale

- Secondary students concepts' for unknowns and variables
  - Letters stand for known rather than indeterminate values.
  - Letters stand for labels of objects rather than quantities
- An increasing number of secondary students take algebra courses (Stein, Kaufman, Sherman, & Hillen, 2011), and so algebra teachers are tasked with working with a greater diversity of students.

Purpose of Paper

- Describe and account for how 13 middle school students across two design experiments conceived of what we call *extensive quantitative unknowns* (EQUs).
- E.g.: What is the relationship between the height of the room measured in toothpick lengths v. skewer lengths?
- Research questions:
  - How did the students conceive of EQUs?
  - How can we account for their conceptions?

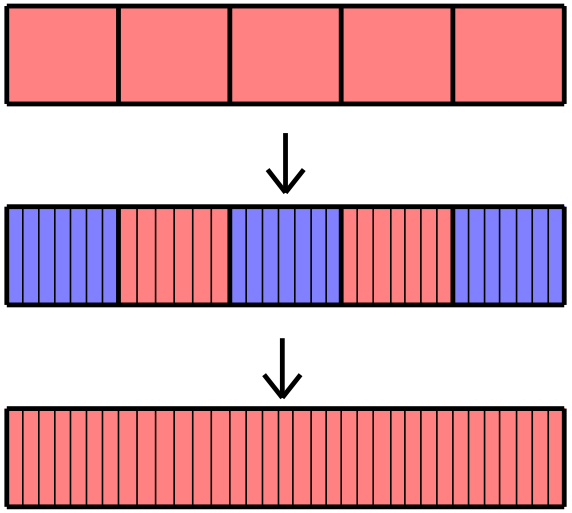
Algebra from a Quantitative Perspective

- Extensive quantities can be directly counted or measured (e.g., distance).
- An *Extensive Quantitative Unknown (EQU)* is an extensive quantity for which a value is not known, but for which a value could be determined.

- Thinking of an EQU requires being able to imagine a unit of units.

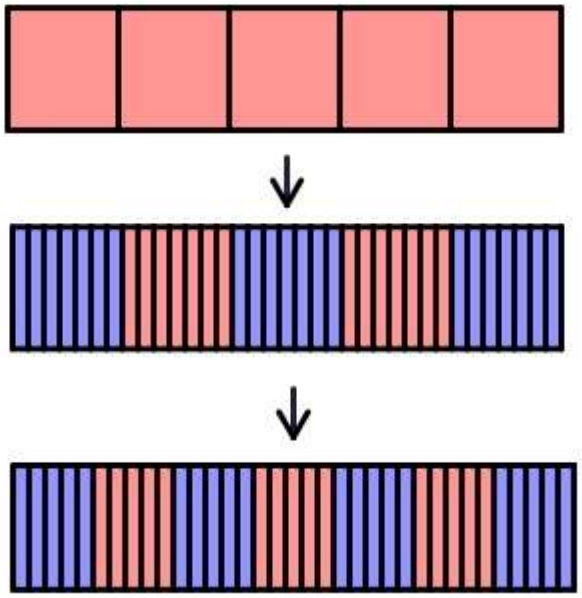
Second Multiplicative Concept (MC2 students)

- Can anticipate the coordination of two levels of units prior to operating
- Can produce three levels of units in activity



Third Multiplicative Concept (MC3 students)

- Can take three levels of units as given
- Can flexibly switch between three-levels-of-units structures

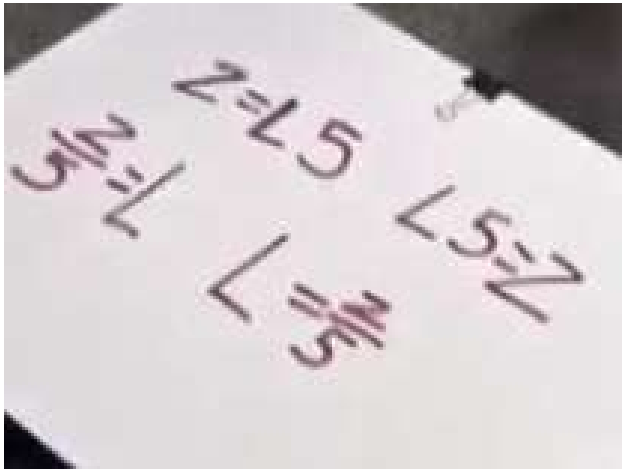


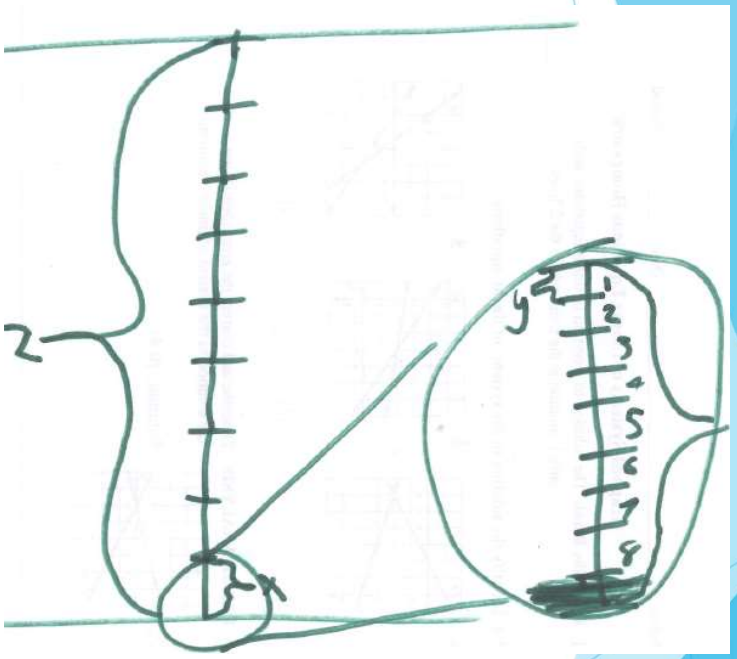
Single Unknown Problems

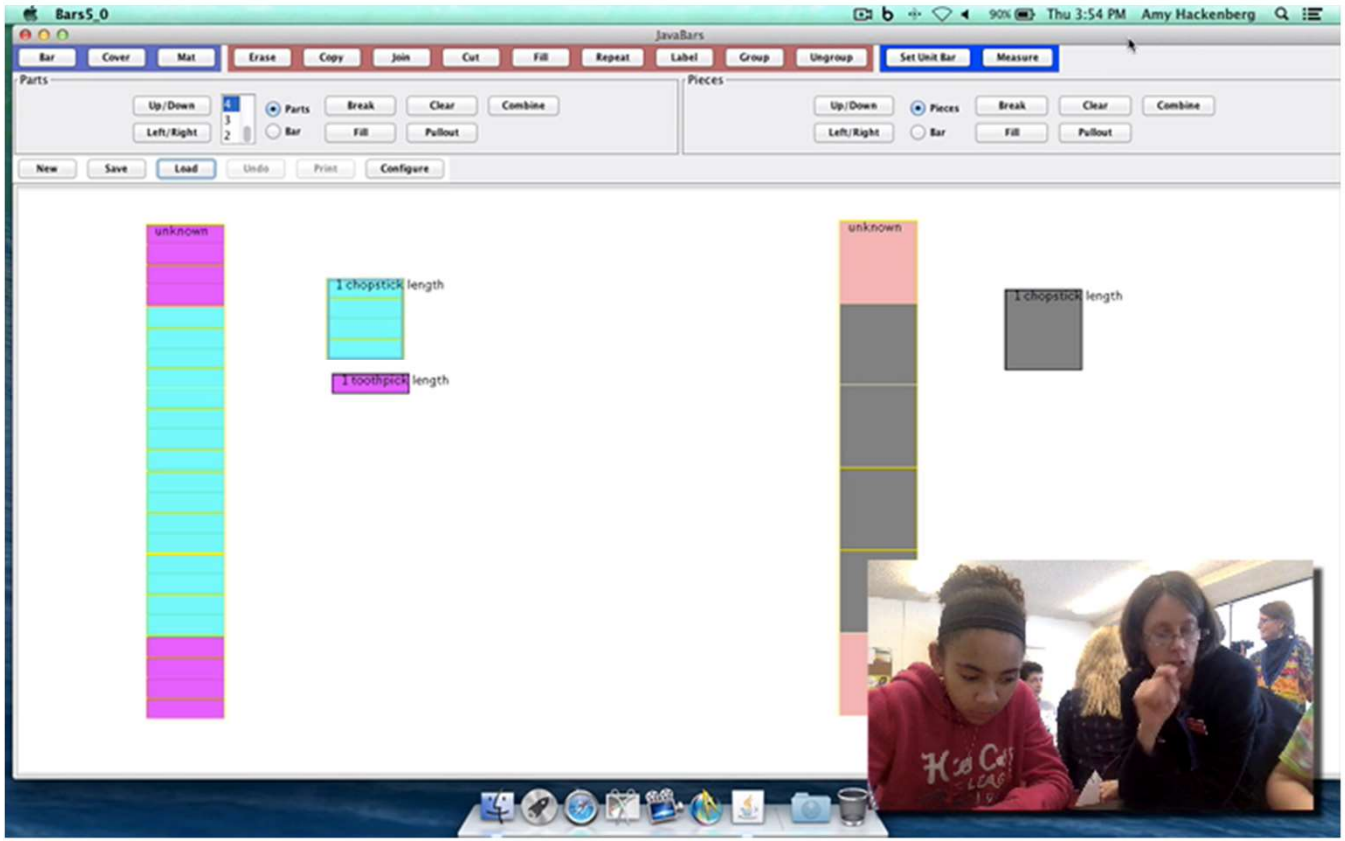
- The height of Tri-North Middle School is unknown. You have two measuring units, a toothpick length and a skewer length. Five toothpick lengths fit into a skewer length. Imagine measuring the school's length in toothpicks, and then measuring it again in skewer lengths. You would get two different values.
  - Draw a picture in JavaBars to show the two values for the height of the school.
  - What equation can you write to show the relationship between these two unknowns? Explain what your equation means in terms of your picture.

Emmett (MC3)

- They chose z and L to stand in for the number of toothpick lengths that fit into the school's height, and for the number of skewer lengths that fit into the school's height, respectively.
  - Teacher: Are *z* and *L* equal?
  - Emmett: The number of toothpick lengths will be 5 times more than the number of skewer lengths.



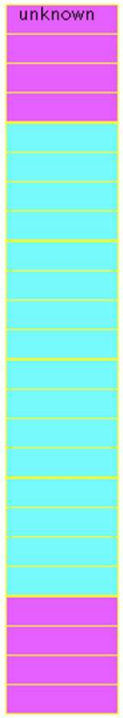


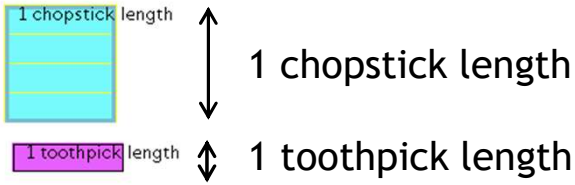


- Defined *C* as “Chopsticks” and *T* as “Toothpicks.”
- Wrote *C* x 4= *T*

Heather (MC2)

“Unknown” [height of school]





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Findings about EQUs:

- We found qualitative differences in how MC2 and MC3 students represented and thought about EQUs.
- MC2 students:
  - Often did not represent the two values for the unknown height accurately (from our perspective) in pictures or equations.
  - Demonstrated conflations that to us indicate their equations did not reflect embedded measurement units.
- MC3 students
  - Sometimes also developed incorrect equations (from our perspective) but revised their work upon questioning.
  - Demonstrated that the equations did reflect embedded units.

Thank you!

- With BIG thanks to others on the **IDR²eAM** project team: Mark Creager, Anna Dinndof, Ayfer Eker, Kemol Lloyd, Rob Matyska, Serife Sevis
- *What IDR²eAM stands for:*
  Investigating Differentiated Instruction and Relationships between Rational Number Knowledge and Algebraic Reasoning in Middle School
- http://www.indiana.edu/~idream/

Implications…

- We need to understand students' measuring schemes and unit structures in order to build algebraic reasoning on quantitative reasoning.
- We also need to develop different supports for MC3 and MC2 students.

IDR²eAM Project: Investigating Differentiated Instruction and Relationships between Rational Number Knowledge and Algebraic Reasoning in Middle School

- **Study:**
  - how to differentiate instruction for cognitively diverse middle school students
  - how students' rational number knowledge and algebraic reasoning are related
- **Phase I (Yrs 1 & 2):** Conducted three 18-episode after school design experiments with 6-9 cognitively diverse middle school students [ages 12-14]