

# Differentiating linear function instruction for 8<sup>th</sup> grade students

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# Rationale: Why differentiate?

- Tracking:
  - Traditional way to deal with student diversity in the US
  - Has negative effects on students and create opportunity gaps (Flores, 2007; Oakes, 2005) that can result in achievement gaps
- Differentiation:
  - An alternative way to address students' diverse learning needs, but largely untested in Math Ed
  - Rare (or absent) in secondary math classrooms (Gamoran & Weinstein, 1998)

# Research on Differentiation

- Gearhart & Saxe (2014):
  - 4<sup>th</sup> & 5<sup>th</sup> grade intervention focused on developing students' understanding of number lines
  - Flexible class structure, problems with multiple entry points, partner work tiered for difficulty
  - Significantly greater learning gains than students in comparison classrooms
- Tieso (2006):
  - Quasi-experimental study with 31 4<sup>th</sup> & 5<sup>th</sup> grade teachers and 645 students, focused on data representation & analysis unit
  - Statistically significant higher scores for regularly-achieving and high-achieving students; low-achieving students' increased scores but not significantly
- Other findings in other fields: positive influence on various aspects of reading (fluency, comprehension) and on students' self-concepts

# IDR<sup>2</sup>eAM Project

Investigating Differentiated Instruction and Relationships between Rational Number Knowledge and Algebraic Reasoning in Middle School

- **Purposes:**

- Study how to differentiate instruction for cognitively diverse middle school students
- Study how students' rational number knowledge and algebraic reasoning are related

# Differentiation, to us

**Definition:** proactively tailoring instruction to students' mathematical thinking while aiming to develop a cohesive classroom community (cf. Tomlinson, 2005)

- Implement on-going assessment to get to know students' thinking
- Continually explore and clarify learning goals for students
- Provide choices
- Use flexible grouping for different purposes
- Establish norms (e.g., re-think fairness)
- Interact responsively during class meetings (Dyer & Sherin, 2015; Jacobs & Empson, 2015)
- Use thinking from individuals and small groups to shape whole classroom discussions (Fennema et al., 1996; Jacobs & Empson, 2015; Leatham et al., 2015; Tomlinson, 2005)

# Overview of IDR<sup>2</sup>eAM Project

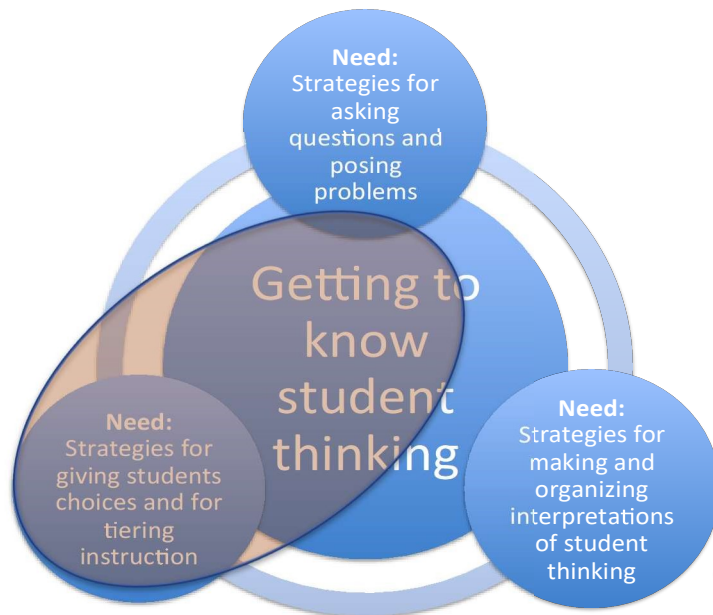
- **Phase I (2 yrs):** Three design experiments after school
  - 6-9 7<sup>th</sup> and 8<sup>th</sup> grade students selected for cognitive diversity
  - 18 episodes each
- **Phase II (1 yr):** Teacher Study Group (TSG) with 15 middle school teachers from around Indiana
  - Summer workshop, 8 monthly meetings, summer workshop
- **Phase III (2 yrs +):** Two design experiments co-taught with teachers who participated in the TSG
  - 26-27 day units using Connected Mathematics Project materials (CMP3)
  - 20-21 students in each class

# Purpose of the talk

**Research Question:** What cognitive and affective influences did tiering instruction have on a class of 20 eighth grade pre-algebra students during a 4-day instructional segment focused on linear functions?

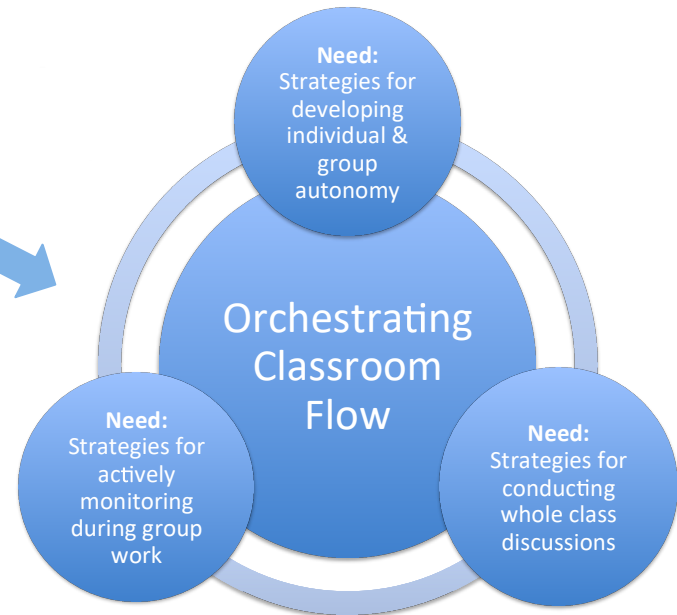
- **Tiering Instruction:** providing different activities/problems to groups of students based on formative assessment of students' ways of thinking
- This segment occurred on Days 20-23 of a 27-day unit on equivalence, using the “Say It With Symbols” materials from 8<sup>th</sup> grade CMP3

# Our emerging theory of differentiating mathematics instruction



We are here today

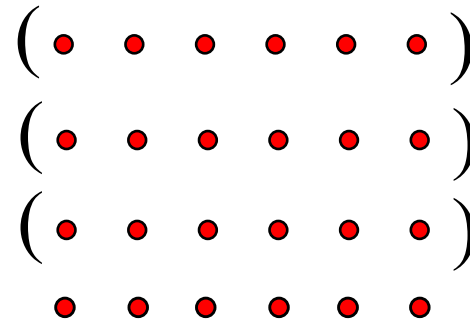
Working Models





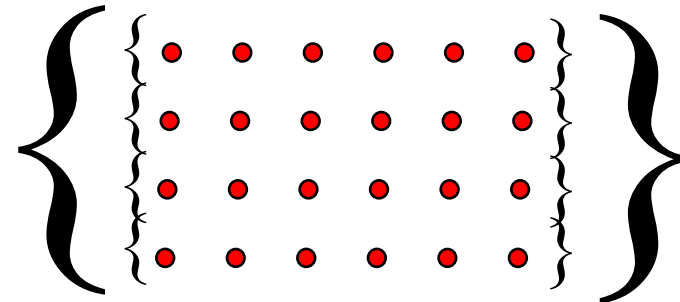
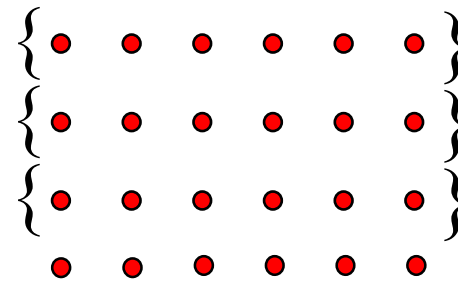
# Units Coordination

- **Composite unit:** a unit of units.
- **Units coordination:** distribute the elements of one composite unit across the elements of another composite unit.
- Three stages of levels of units coordination:
  - Influences fraction knowledge, proportional reasoning, equation writing, combinatorial reasoning...
  - Transition between them can be protracted (Steffe & Cobb, 1998; Steffe & Olive, 2010)
- **Problem:** There are 18 flower pots. Each one can be filled with 6 pounds of soil. How much soil is needed to fill all of them?
- **Stage 1:** 6 is a composite unit
  - But, no multiplicative relationship between 1s and the 6
  - Can track groups of 6s and 1s in activity, often counting on by 1s past known skip-counting patterns



# Units Coordination

- **Stage 2:** 6 is a composite unit with a multiplicative relationship between 1s and 6
  - Takes the coordination of 18 6s as given and can break apart 6s to reason with them
- **Stage 3:** 6 is a composite unit and iterations of 6, such as 24, are composite units of composite units
  - Takes coordination of 18 6s as a three-levels-of-units structure
  - Can reason with 6s as if 1s but not lose them as 6s
  - Can switch to view the 24 as six 4s



**Crate Problem:** There are 4 cans of juice in a package and 8 packages in a box. A crate contains 6 boxes. How many cans of juice are in a crate, and can you draw a picture to show how you know?

Units Coordination Framework		
Stage 1	Students can take one level of units as given, and may coordinate two in activity.	Often must “build up from ones” to nest quantities, and cannot keep multiple levels in mind when operating further.
Stage 2	Students can take two levels of units as given, and may coordinate three in activity.	They can iterate composite units, so a package can be both 1 package and 4 cans even as they’re building up 8 of them into a box. Sometimes conflate boxes and packages when working with a crate.
Stage 3	Students take three levels of units as given, and can flexibly switch between three level structures.	They can usually move flexibly among packages, boxes and crate without conflation.

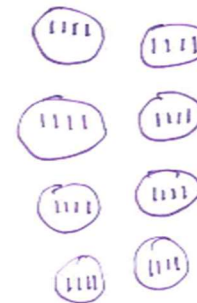
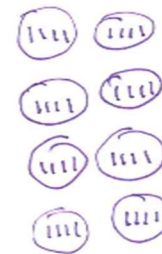
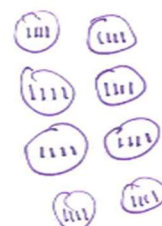
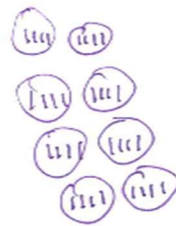
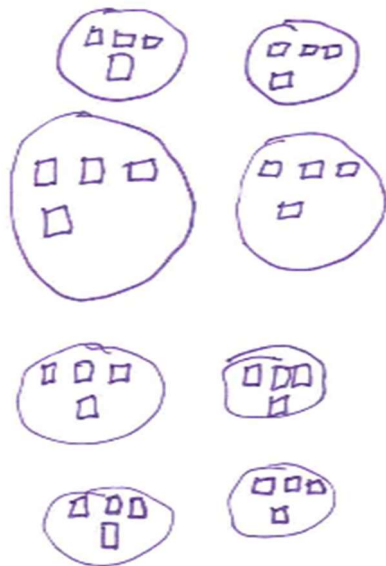
There are 4 cans of juice in a package and 8 packages in a box. A crate contains 6 boxes.  
How many cans of juice are in a crate, and can you draw a picture to show how you know?

## Units Coordination - Alyssa

Stage 1

Students can take one level of units as given, and may coordinate two in activity.

Often must build up from ones to nest quantities, and cannot keep multiple levels in mind when operating further.



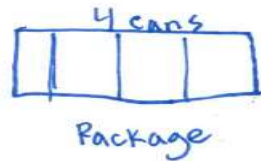
There are 4 cans of juice in a package and 8 packages in a box. A crate contains 6 boxes.  
How many cans of juice are in a crate, and can you draw a picture to show how you know?

## Units Coordination – Joanna

Stage 2

Students can take two levels of units as given, and may coordinate three in activity

They can iterate composite units, so a package can be both a package and 4 cans even as they're building up 8 of them into a box. Sometimes they conflate boxes and packages when working with a crate.

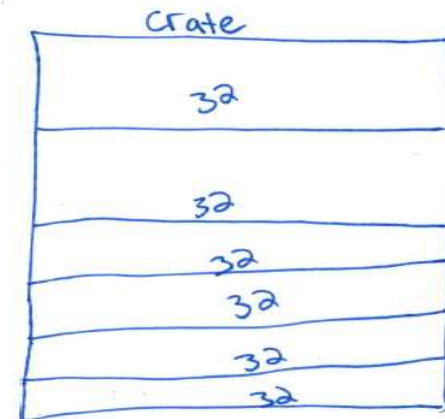
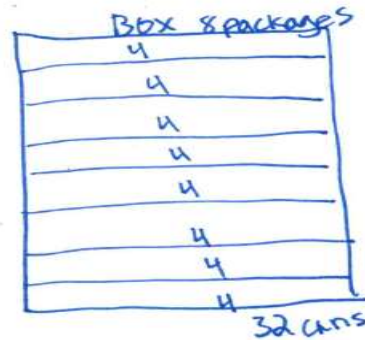


AH: Okay. So are a box and a package the same? Or are they different?

J: They're the same.

AH: The same. Okay.

J: Well, like kind of. In a box there's 8 cans, or 8 sections of 4, versus 6 sections of 32.



$$\begin{array}{r} 32 \\ \times 6 \\ \hline 192 \text{ cans} \end{array}$$

# Spring 2017 design experiment

- Participating classroom: 8<sup>th</sup> grade pre-algebra, 20 students
- Selected two other classrooms for comparison: 23 students
- Gathered initial information:
  - Initial written assessments on units coordination and fractions knowledge
  - Individual interviews with 31 students
- Results:

Units Coordination Level	Participating Class	Comparison Classes
Stage 1	5	3
Stage 2	13	15
Stage 3	2	5

- Selected 6 participating focus students (two stage 1, two stage 2, two stage 3) and 6 comparison focus students matched on units coordination and aspects of fraction knowledge

# Data Collection and Analysis

**The Unit, Say It With Symbols: writing and combining equivalent expressions, solving equations, **writing expressions to represent linear relationships****

## **Data:**

- Daily: whole-class and small group video, copies of written student work
- Middle of Unit – 6 focus students from participating class
- End of the Unit – 12 focus students – participating and comparison

## **Analysis:**

Development of second-order models of student thinking – analysis of all data sources, discussions with research team

# Example of tiering instruction

Section 4.1 of Say it With Symbols introduces the following situation:

Magnolia Middle School needs to empty their pool for resealing. Ms Theodora's math class decides to collect data on the amount of water in the pool and the time it takes to empty it.

The class writes the following equation to represent the amount of water  $w$  (in gallons) in the pool after  $t$  hours.

$$w = -250(t - 5)$$

- What information does the  $-250$  represent?
- What units should you use for  $-250$ ?
- What information does  $(t - 5)$  represent? What units should you use for  $(t - 5)$ ?
- What units should you use for  $-250(t - 5)$ ? Explain.



# Tiering the Pool Problem

## Version A

2) Pauline comes in at noon to fill the pool. There are already 1080 gallons of water in it. She fills it at 12 gal/min:

$$w = 12t + 1080$$

## Version B

3) Pauline comes in at noon to fill another pool, and the pool is empty. She gets a phone call and does not start filling the pool for 5 minutes. She fills it at 12 gal/min:

$$w = 12(t - 5)$$

g) An engineer comes in partway through the day and figures out how many gallons are in the pool. She does it again 15 minutes later. How much will the amount of water change in that 15 minutes? Does it matter when during the day she arrives?

# Current Findings: Overview

- Tiering instruction seemed effective because students' work revealed conceptual difficulties with the delayed start time.
- Some students made progress cognitively.
- Tiering had an affective effect on some.

# Summary of Days 20-23

#1) Worker filling an empty pool starting at noon, using a hose that flowed at a rate of 12 gallons per minute

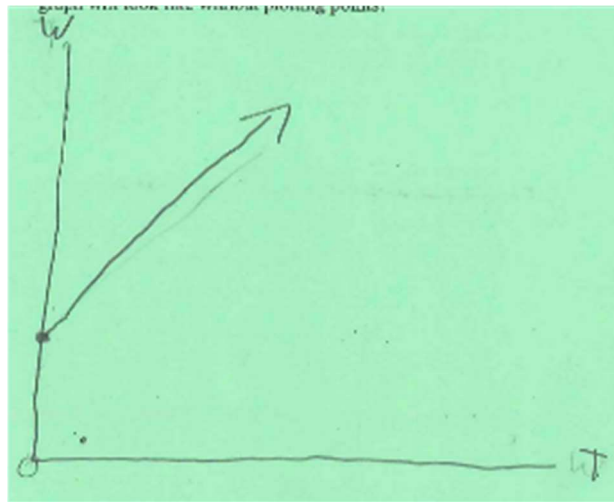
- Day 20
  - Task 1 introduced
  - Whole class discussion then small group work
- Day 21
  - Each small group given one of two tiered tasks (#2 and 3)
- Day 22
  - Continued group work
  - Whole class discussion of both tasks, including
  - Each group given one of two tiered tasks for homework (#4 and 5)
- Day 23
  - Small group work
  - Whole class discussion of 4 and 5, focused on graphs

# Darrin and Kathy

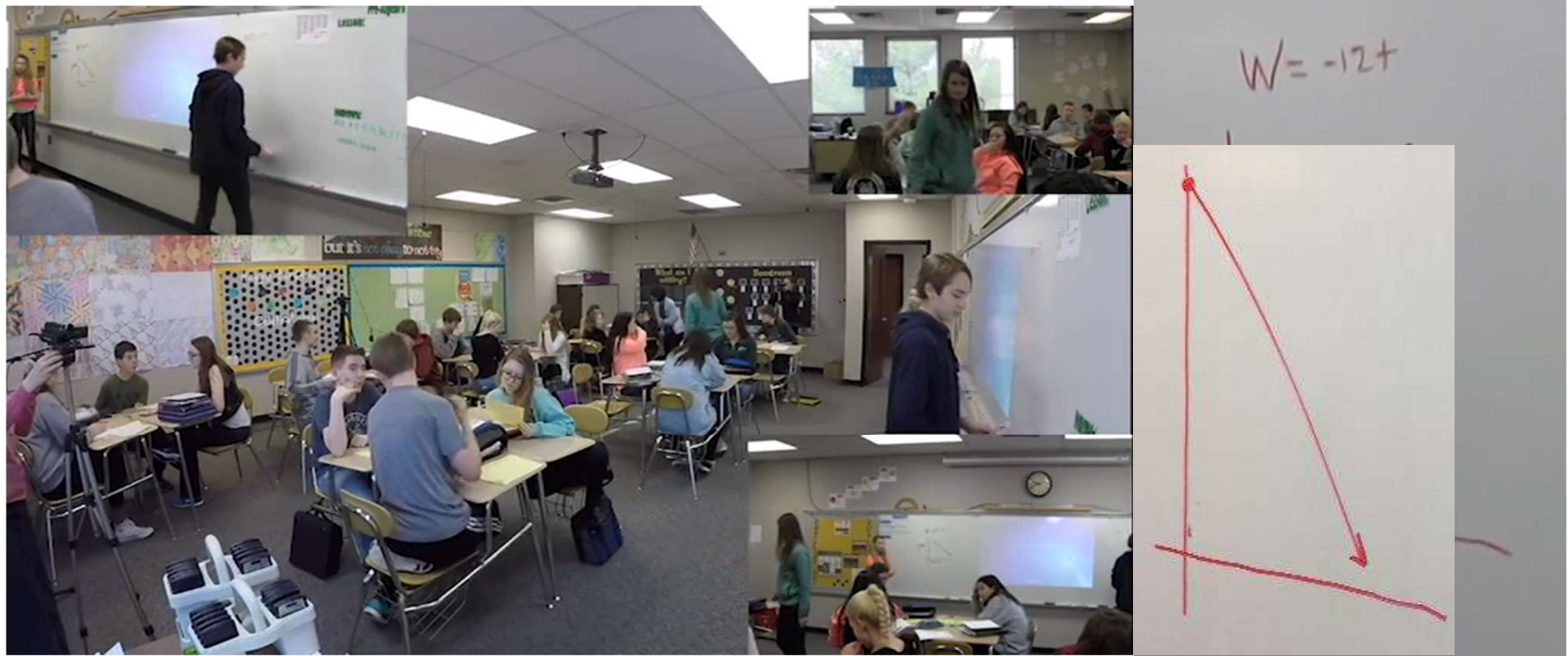
We want to show you now two examples of students with very different ways of thinking, both working at their edge.

- The big idea: we wanted students to think about the coordination of both quantities (time, water in pool) as they were changing.
- DARRIN: in interviews and school work, he struggled to coordinate multiple quantities but did well connecting context to representations and could simplify situations to help himself be successful
- KATHY: Didn't see herself as good at math but was very engaged in class, thought deeply and asked great questions.

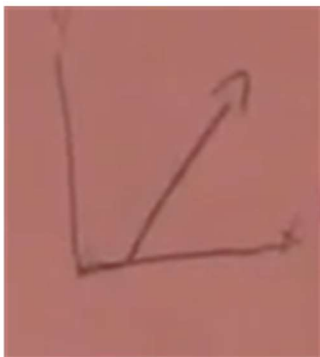
**Darrin: Task 2,  $w = 12t + 1080$**



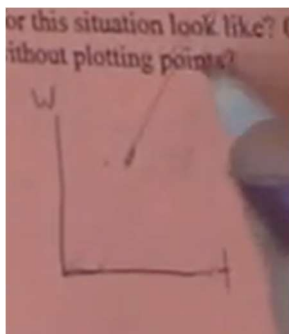
# Darrin: Task 4, $w = -12t$



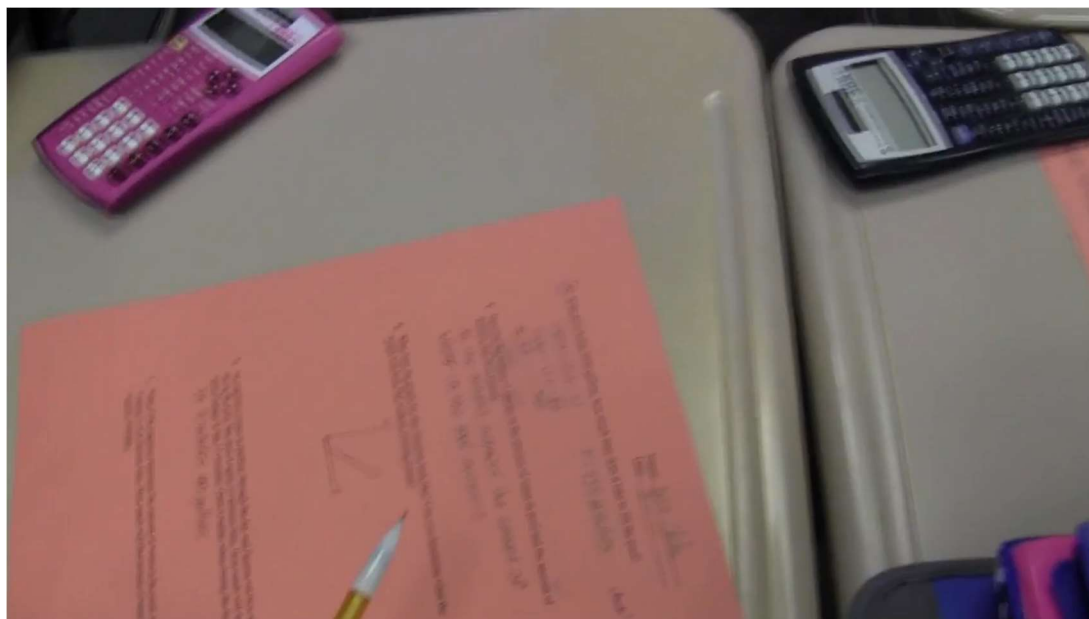
# Kathy



Kathy's initial graph for  
 $w = 12(t - 5)$



Second attempt, 'starting' at 12 gallons, 6 minutes



# Affective Influences

- CONCERNS

- Several mentioned the possibility of students feeling ‘dumb’, but only one was a comment in the first person.
- Lack of flexibility. Don’t hold them back if they’re ready to move on.
- Group dynamics – one student didn’t want to feel more or less affinity for others based on how they were grouped.

- OPTIMISM

- Student involvement might increase with greater match between level of understanding and level of assignments
- Both struggling and advanced students could have the work they need.
- Recognition that students are already aware of cognitive differences. This just addresses them.
- Enthusiasm for learning from those who thought differently.



# Thank you!

- With BIG thanks to other members of the **IDR<sup>2</sup>eAM** project team: Ayfer Eker, Mark Creager, Sharon Hoffman, Serife Sevis, Pai Suksak
- *What IDR<sup>2</sup>eAM stands for:* Investigating Differentiated Instruction and Relationships between Rational Number Knowledge and Algebraic Reasoning in Middle School
- <http://www.indiana.edu/~idream/> ← paper and references will be available here

# Future Work

- Paper in preparation about our theory of differentiation based on Phase I & II
- Continued analysis of data from the two experiments in Phase III
- Paper in preparation based on the findings shared here today
- Proposal to present at 2019 NCTM meeting in collaboration with teachers from the two experiments in Phase III