Differentiating Linear Function Instruction for Eighth Grade Students

from the IDR²eAM Project: **I**nvestigating **D**ifferentiated Instruction and **R**elationships between **R**ational Number Knowl**e**dge and **A**lgebraic Reasoning in **M**iddle School, funded by the National Science Foundation. http://www.indiana.edu/~idream/

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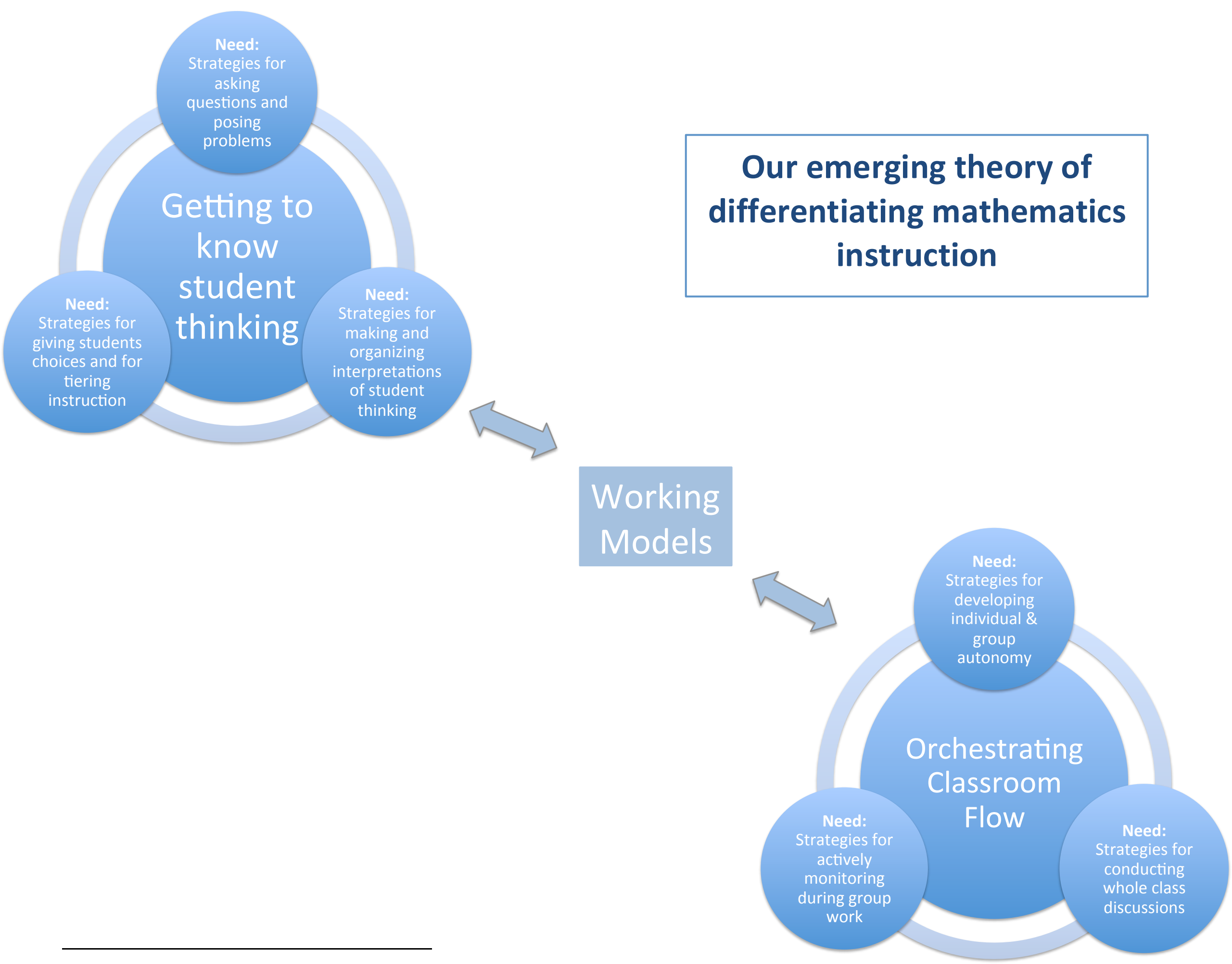
Differentiating instruction is a pedagogical approach to manage classroom diversity in which teachers proactively plan for adapting curricula, teaching methods, student activities, and products of learning to address individual students' needs in an effort to maximize learning for all.[1]

Our working definition: proactively tailoring instruction to students' mathematical thinking while aiming to develop a cohesive classroom community.



[1] Tomlinson, C. A. (2005). *How to differentiate instruction in mixed-ability classrooms* (2nd ed.). Upper Saddle River, NJ: Pearson.

Students' Units Coordination Levels for spring 2017 design experiment
as assessed at the beginning of the experiment through written assessments and interviews

Units Coordination Level	Participating Class	Comparison Classes
Stage 1	5	3
Stage 2	13	15
Stage 3	2	5

One task used to assess students' units coordination levels:

Crate Problem: There are 4 cans of juice in a package and 8 packages in a box. A crate contains 6 boxes. How many cans of juice are in a crate, and can you draw a picture to show how you know?

One DI strategy used during the experiments: Tiered Instruction
Our Definition of Tiered Instruction: providing different activities and/or problems to groups of students based on formative assessment of students' ways of thinking.

Investigation 4.1 from "Say It With Symbols" unit in Connected Mathematics Project materials[2]

Original problem (pp. 66-67)
"Magnolia Middle School needs to empty their pool for resealing. Ms. Theodora's math class decides to collect data on the amount of water in the pool and the time it takes to empty it. The class writes the following equation to represent the amount of water w (in gallons) in the pool after t hours.

$$w = -250(t - 5)$$

Our Tiered Versions

All students did the following problem:

1) Pauline the pool worker comes in to fill a pool at noon on Monday. Her hose fills the pool at 12 gallons per minute.
    a. What equation could you write to represent the amount of water in the pool, w, after t minutes since noon?
    b. How many gallons will be in the pool after 30 minutes? An hour? 90 minutes?
    c. If the pool holds 5400 gallons, how much time does it take to fill the pool?
    d. Describe the pattern of change in the amount of water in the pool and the amount of minutes that have passed.
    e. What does the graph for this situation look like? Can you determine what the graph will look like without plotting points?

Then each student was assigned one of the following:

[2] Lappan, G., Phillips, E. D., Fey, J. T., Friel, S. N., Grant, Y., & Stewart, J. (2014). *Connected mathematics 3*. Boston, MA: Pearson.

<table>
<tr><th>Version A</th><th>Version B</th></tr>
<tr><td>Remember Pauline the pool worker? She is still working at her job maintaining pools.

2) When Pauline comes in at noon on Tuesday to fill another identical pool, fortunately she finds that there are already 1080 gallons of water in the pool. She will still fill the pool at a rate of 12 gallons per minute. This equation expresses the amount of water, w, in the pool after t minutes since noon:

$w = 12t + 1080$

a. What does the 12 mean in the equation?
b. Why is 1080 added to $12t$ in the equation?
c. How many gallons will be in the pool after 30 min? After an hour?
d. If the pool holds 5400 gallons, how much time does it take to fill the pool?
e. Describe the pattern of change in the amount of water in pool and the amount of minutes that have passed.
f. What does the graph for this situation look like? Can you determine what the graph will look like without plotting points?
g. An engineer comes in partway through the day and figures out how many gallons are in the pool. She does it again 15 minutes later. How much will the amount of water change in that 15 minutes? Does it matter when during the day she arrives?</td><td>Remember Pauline the pool worker? She is still working at her job maintaining pools.

3) When Pauline comes in at noon on Wednesday to fill another identical pool, the pool is empty. She gets a phone call and does not start filling the pool for 5 minutes. This equation expresses the amount of water, w, in the pool after t minutes since noon:

$w = 12(t - 5)$

a. What does the 12 mean in the equation?
b. Why is 5 subtracted from t in this equation?
c. Write an expression for w that is equivalent to the original expression in the equation above. What information does this new expression tell you about the situation?
d. How many gallons will be in the pool after 30 min since noon? After an hour since noon?
e. If the pool holds 5400 gallons, how much time does it take to fill the pool?
f. Describe the pattern of change in the amount of water in pool and the amount of minutes that have passed.
g. What does the graph for this situation look like? Can you determine what the graph will look like without plotting points?
h. An engineer comes in partway through the day and figures out how many gallons are in the pool. She does it again 15 minutes later. How much will the amount of water change in that 15 minutes? Does it matter when during the day she arrives?
i. What if the engineer measures the amount of water in the pool, and then measures it again ½ of a minute later. How much will the amount of water change in that ½ minute? Explain.</td></tr>
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<tr><th>Version A</th><th>Version B</th></tr>
<tr><td>4) When Pauline comes in at noon, a pool identical to the one on the other days is full. She is supposed to empty the pool, and it empties at 12 gallons per minute. This equation expresses something about the amount of water in the pool after t minutes since noon:

$w = -12t$

a. Explain why the equation involves -12. What does that mean?
b. When t is 30, w is -12(30)=-360. What do the values of t = 30, w = -360 mean in the situation?
c. The pool holds 5400 gallons. Find out how much water is left in the pool 2 hours after noon.
d. Describe the pattern of change in the amount of water in the pool and the amount of minutes that have passed.
e. What does the graph for this situation look like? Can you determine what the graph will look like without plotting points?
f. An engineer measures the amount of water in the pool, and then measures it again ½ minute later. How much will the amount of water change in that ½ minute? Does it matter which ½ minute she measures during the day?</td><td>5) When Pauline comes in at noon on Friday, a pool identical to the one on the other days is full. She is supposed to empty the pool, and it empties at 12 gallons per minute. However, first she has to change the filter for the pool, which takes 10 minutes. This equation expresses something about the amount of water in the pool after t minutes since noon:

$w = -12(t - 10)$

a. Explain why the equation involves -12. What does that mean?
b. Why is 10 subtracted from t in this equation?
c. Write an expression for w that is equivalent to the original expression in the equation above. What information does this new expression tell you about the situation?
d. When t is 30, w is -12(30-10)=240. What do the values t = 30, w = 240 mean in the situation?
e. The pool holds 5400 gallons. Find out how much water is left in the pool 2 hours after noon.
f. Describe the pattern of change in the amount of water in the pool and the amount of minutes that have passed.
g. What does the graph for this situation look like? Can you determine what the graph will look like without plotting points?
h. What is the y-intercept for this graph? What could the y-intercept mean in this situation?</td></tr>
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Transcript for Kathy's video, focused on #3, $w = 12(t - 5)$

Researcher: So what does that point mean [where Kathy's graph crosses horizontal axis]?
Angela: Five [minutes after noon].
Kathy: No, six, so that would be 12:06 because she wasted 5 minutes.
Researcher: Right where that hits right there, what is *w* [gallons of water in pool]?
Kathy: Zero…twelve, it's twelve.
Researcher: Right down here, right at this point [points to Angela's paper, where t = 5, w = 0]?
Angela: That's zero.
Kathy: No, at six it would be twelve.
Researcher: At six it would be twelve, but you're saying at this point where it intersects, it would be zero?
Kathy: I'm confused.
Researcher: If I just plotted this point right here--
Kathy: Oh wait, no, that's not correct. It would have to be up higher…