

# CRATE PROBLEM

**Please take a couple minutes to work on the “Crate Problem” and “Partial Crate Problem” on your handout.**

**If you have time before we start, discuss it with those around you and consider how middle school students might explain their work on this task.**

# TWO HEADS ARE BETTER THAN ONE

Differentiating  
instruction  
through a  
teacher-researcher  
partnership



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Robin Jones & Rebecca Borowski  
Doctoral Students, IU Bloomington

Marie Johannisson  
Teacher, Batchelor Middle School,  
Bloomington, IN

Amy Hackenberg  
Associate Instructor, IU Bloomington

**Who  
We Are**

# OUR GOAL TODAY

- Share lessons learned from a teacher-researcher partnership to study differentiated instruction (DI) in middle school math classes.

The IDR<sup>2</sup>EAM project: **I**nvestigating **D**ifferentiated Instruction and **R**elationships between **R**ational Number Knowledge and **A**lgebraic Reasoning in **M**iddle School.

- 5 years, 3 phases
  - Phase 1 (2013-2015) – After School Math Classes
  - Phase 2 (2015-2016) – Teacher Study Group
  - Phase 3 (2016-2018) – DI in full classes

# DIFFERENTIATION, TO US

DI, to us, is proactively tailoring instruction to students' mathematical thinking while aiming to develop a cohesive classroom community (cf. Tomlinson, 2005)

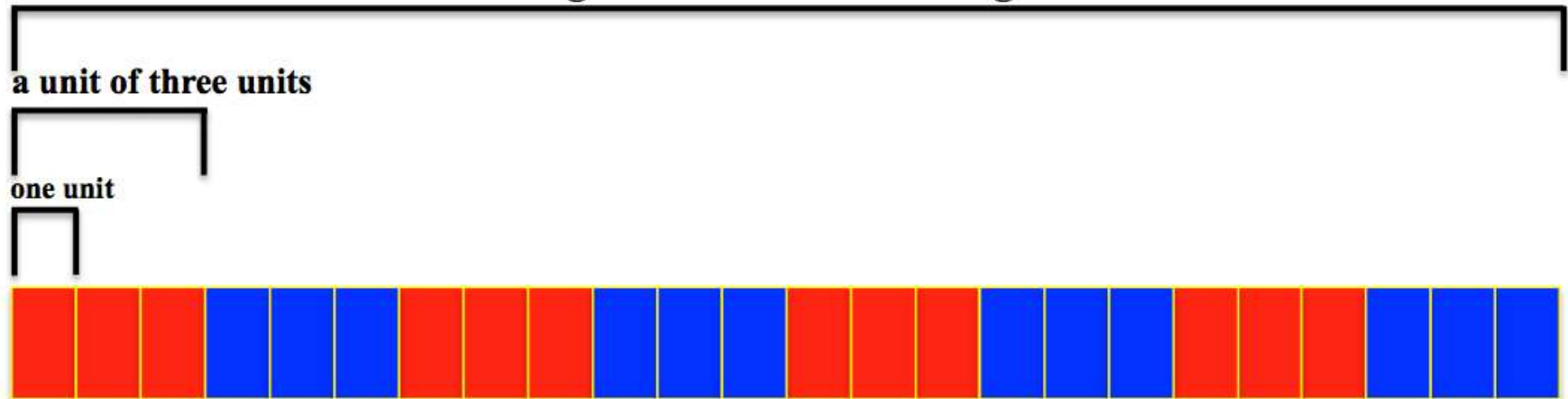
- Posing problems in harmony with and at the edge of students' thinking (Hackenberg, 2010), which relies on on-going formative assessment (Heacox, 2002)
- Interacting responsively during class meetings (Jacobs & Empson, 2015)
- Seeking to use thinking from individuals and small groups to shape whole classroom interactions (cf. Tomlinson, 2005)

# DI STRATEGIES

- **Open Questions** – allow student interpretation of a concept to be visible
- **Choice Questions** – same problem, students choose numbers that are a good level of challenge for them
- **Parallel Tasks** – same concept, different problems. Students choose.
- **Tiering Instruction** – same big idea, different problems. Teachers choose
- **Posing questions to individuals and groups, listening to responses**

# UNITS COORDINATION - OVERVIEW

**a unit of eight units each containing three units**



# UNITS COORDINATION – 3 STAGES

<b>Stage 1</b>	<b>Students can take one level of units as given, and may coordinate two in activity.</b>	<b>Often must “build up from ones” to nest quantities, and cannot keep multiple levels in mind when operating further.</b>
<b>Stage 2</b>	<b>Students can take two levels of units as given, and may coordinate three in activity</b>	<b>They can iterate composite units, so a package can be both a package and 4 cans even as they’re building up 8 of them into a box. Sometimes conflate boxes and packages when working with a crate.</b>
<b>Stage 3</b>	<b>Students take three levels of units as given, and can thus flexibly switch between three level structures</b>	<b>They can usually move flexibly among packages, boxes and crate without conflation.</b>

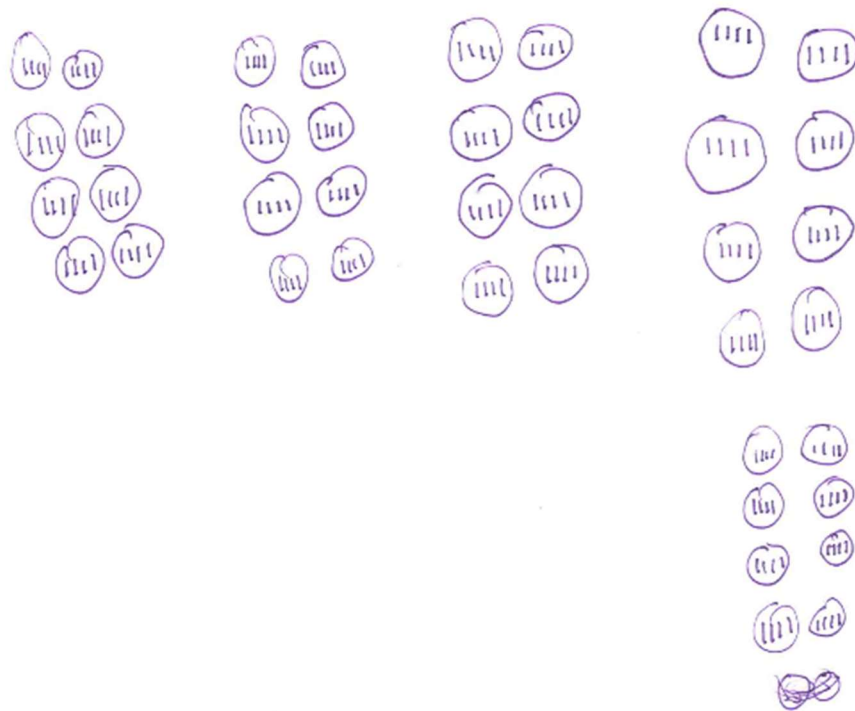
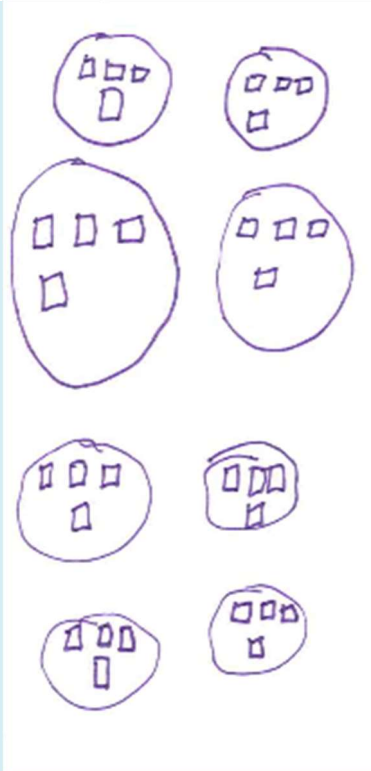


# UNITS COORDINATION - ALYSSA

## Stage 1

Students can take one level of units as given, and may coordinate two in activity.

Often must “build up from ones” to nest quantities, and cannot keep multiple levels in mind when operating further.

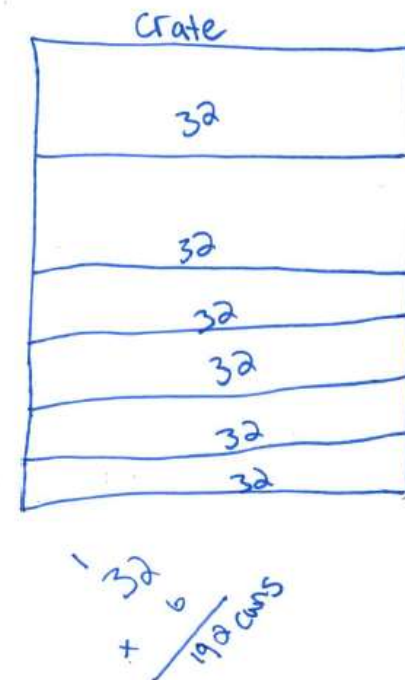
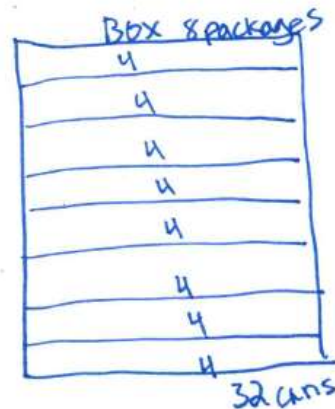
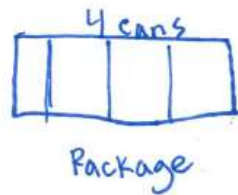


# UNITS COORDINATION – JOANNA

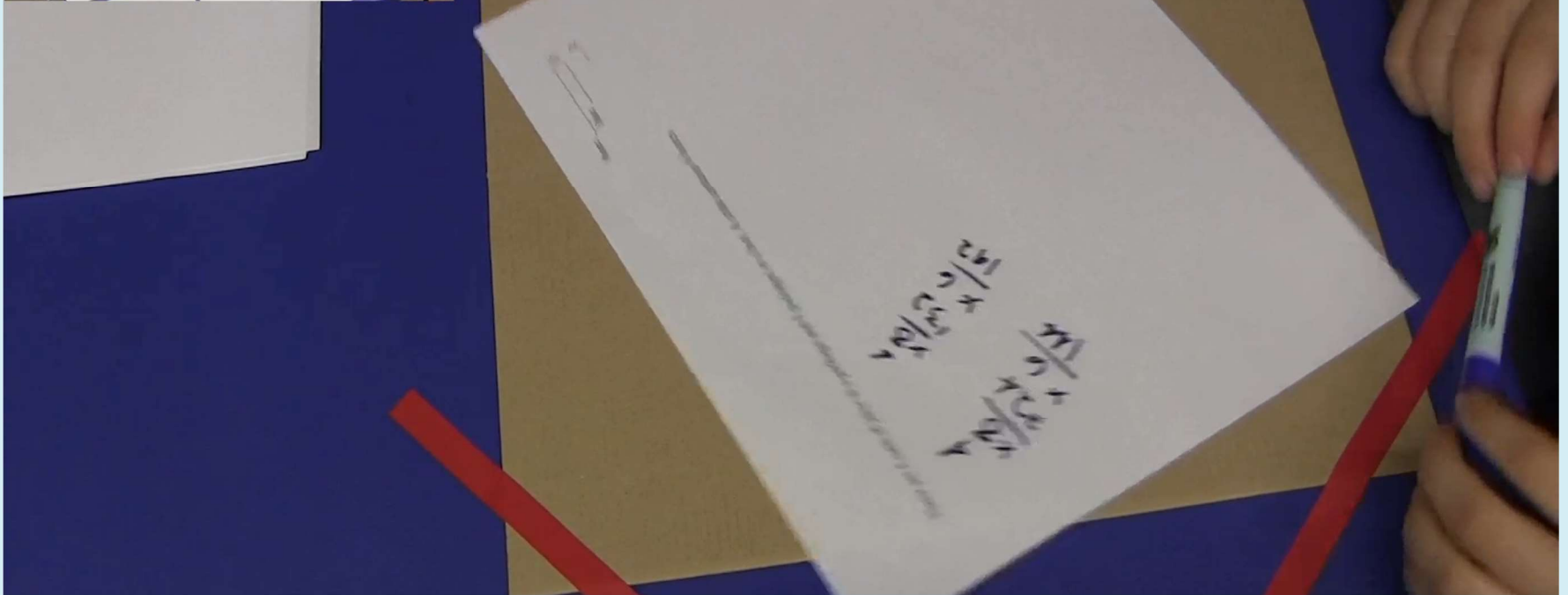
## Stage 2

Students can take two levels of units as given, and may coordinate three in activity

They can iterate composite units, so a package can be both a package and 4 cans even as they're building up 8 of them into a box. Sometimes conflate boxes and packages when working with a crate.



# UNITS COORDINATION – ISABELLE



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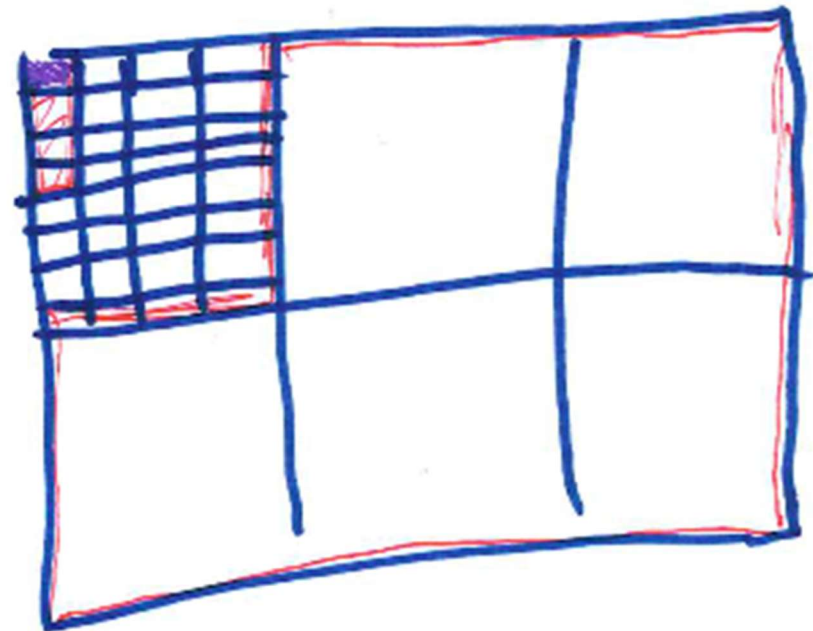
Stage 3

Students take three levels of units as given, and can thus flexibly switch between three level structures

They can usually move flexibly among packages, boxes and crate without conflation.

$$\begin{array}{r} 4 \\ \times 8 \\ \hline 324 \\ \times 6 \\ \hline 144 \end{array}$$

$$\begin{array}{r} 1 \\ \times 8 \\ \hline 132 \\ \times 6 \\ \hline 142 \end{array}$$



# IDREAM - PHASE 1

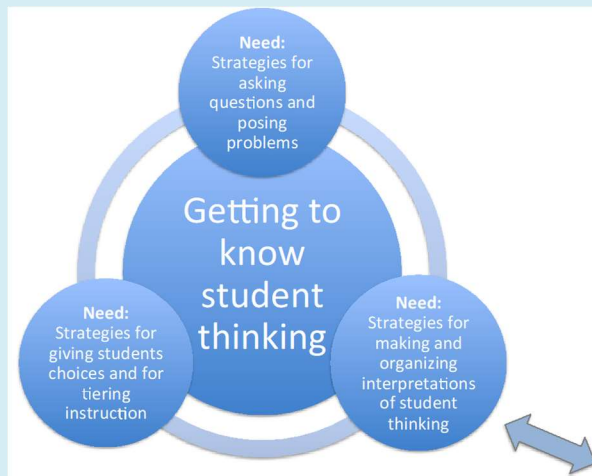
## ■ After-school math classes

- 18 sessions – twice a week for 9 weeks, 1 hour each
- 6 – 9 students
- Students volunteered
- Selection based on ensuring a group that was cognitively diverse (specifically, with respect to multiplicative reasoning)

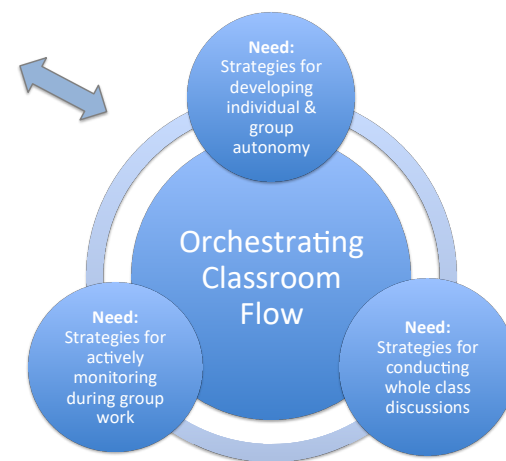
## ■ Differentiation in Phase 1

- Choice problems, open questions, parallel tasks, tiered instruction used to elicit student thinking and address differences
- Heterogeneous and Homogeneous grouping used depending on task
- Whole-group discussions were key to classroom cohesion

# OUR THEORY OF DI



## Working Models



# PHASE 2 – TEACHER STUDY GROUP

## ■ Teacher Study Group

- 15 teachers from across Indiana
- Met for 3 days the summer of 2015
- Monthly meetings after school
- Final day to share results from differentiated unit

## ■ Lessons learned

- Student thinking must be at the center
- Eliciting and interpreting student thinking is not a trivial task
- Di does not require wholesale change of structure
  - Start small
  - Keep focused on student reasoning as the motivator for any changes

# PHASE 3

- **Teacher – Researcher partnerships**
  - Spring, 2017: Marie and Amy taught a differentiated version of CMP's "Say it With Symbols" unit.
  - Fall, 2017: 7<sup>th</sup> grade CMP unit currently underway
- **Getting to know student thinking**
  - written assessment
  - initial interviews with many students to assess thinking and find 'focus students'
  - mid-unit and final interviews with focus students
- **Planning and implementing full unit together**
  - teaching the unit to a regular class during the school day
  - planning for differentiation as needed



# SPRING 2017 OVERVIEW

- **Say it With Symbols (CMP 3):**
  - 27 days in class
  - 4 investigations (omitted 5<sup>th</sup> one)
  - 21 8<sup>th</sup> grade pre-algebra students (plus 2 not participating)
    - 6 stage 1, 12 solid stage 2, 2 borderline stage 2 to 3, 1 stage 3
  - First exposure to CMP – they did not have any experience with the 6<sup>th</sup> and 7<sup>th</sup> grade CMP units
- **Differentiation included:**
  - 1.3 – choice task
  - 2.3 – tiering instruction
  - 4.1 – tiering instruction
  - Questioning and supporting small groups throughout the unit

# MARIE'S PERSPECTIVE

Where did differentiating happen?

- Before the lessons (and between lessons)
  - Unit planning – standards & concepts addressed, changes needed
  - Pre-assessment – individual student assessment, grouping strategies
- During the lessons
  - Questioning students about their thinking
  - Strategically talking with certain students
  - Strategically planning student sharing of work
- What continued after the unit was over?
  - Pre-assessments
  - Grouping strategies
  - Tiered lessons

# AMY'S PERSPECTIVE

- Why was it vital to work in partnership with a teacher, in the context of teaching a full class?
- What made Say it With Symbols a good unit for differentiation?
- What did you learn about DI in running this experiment?

# EXAMPLE OF TIERING INSTRUCTION

Section 4.1 of Say it With Symbols introduces the following situation:

Magnolia Middle School needs to empty their pool for resealing. Ms Theodora's math class decides to collect data on the amount of water in the pool and the time it takes to empty it.

The class writes the following equation to represent the amount of water  $w$  (in gallons) in the pool after  $t$  hours.

$$w = -250(t - 5)$$

- What information does the -250 represent?
- What units should you use for -250?
- What information does  $(t - 5)$  represent? What units should you use for  $(t - 5)$ ?
- What units should you use for  $-250(t - 5)$ ? Explain.

# OUR ALTERNATIVES

On your handout you have five questions written as alternatives to the previous problem.

The first one was given to everyone, and gave them very little trouble.

On the left side is a question we consider accessible to students at stage 1. On the right side is one we gave to students at stages 2 and 3.

We'd like you to take a few minutes to work on them.

Questions to consider as you work:

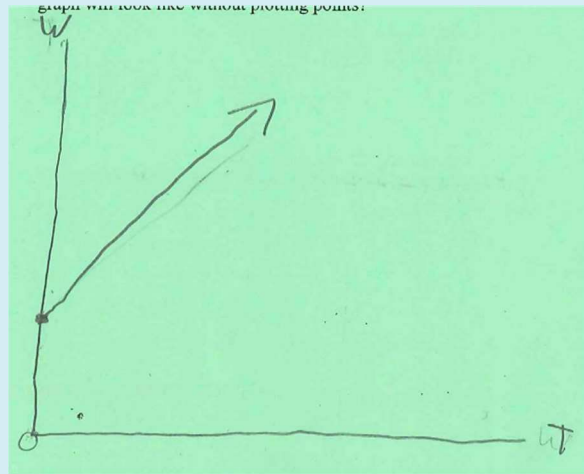
- In what ways are these problems accessible to students with different understanding of variable?
- In what ways might these problems push students to work at the edge of their thinking?

# DARRIN AND KATHY

We want to show you now two examples of students with very different ways of thinking, both working at their edge.

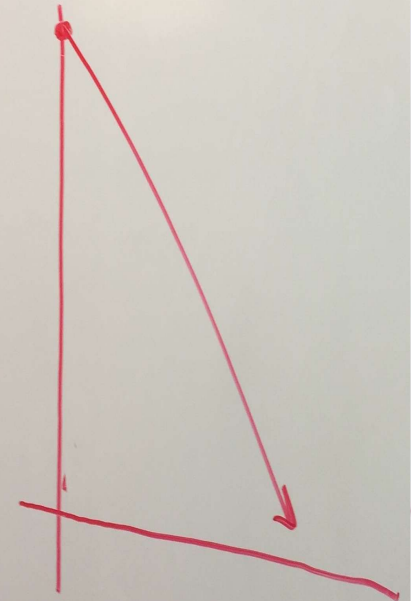
- The big idea: we wanted students to think about the coordination of both quantities (time, water in pool) as they were changing.
  - NOTE: all students in this class could graph equations by creating tables of points. We wanted to push them to think more about relationships between changing quantities.
- DARRIN: in interviews and school work, he struggled to coordinate multiple quantities but did well connecting context to representations and could simplify situations to help himself be successful
- KATHY: Didn't see herself as good at math but was very engaged in class, thought deeply and asked great questions.

# DARRIN



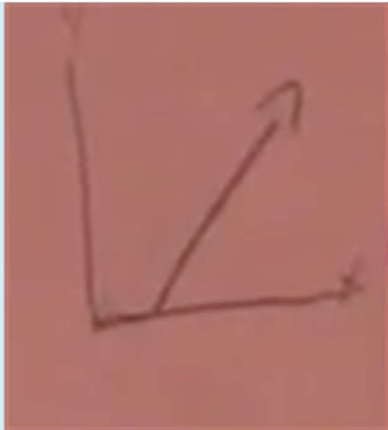
initial graph of  $w = 12t + 1080$

# DARRIN

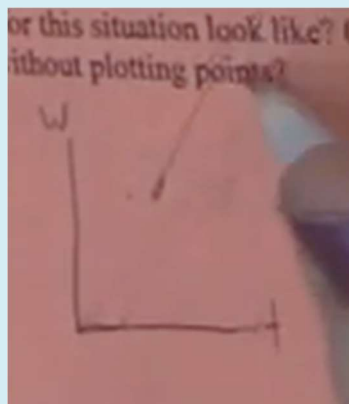




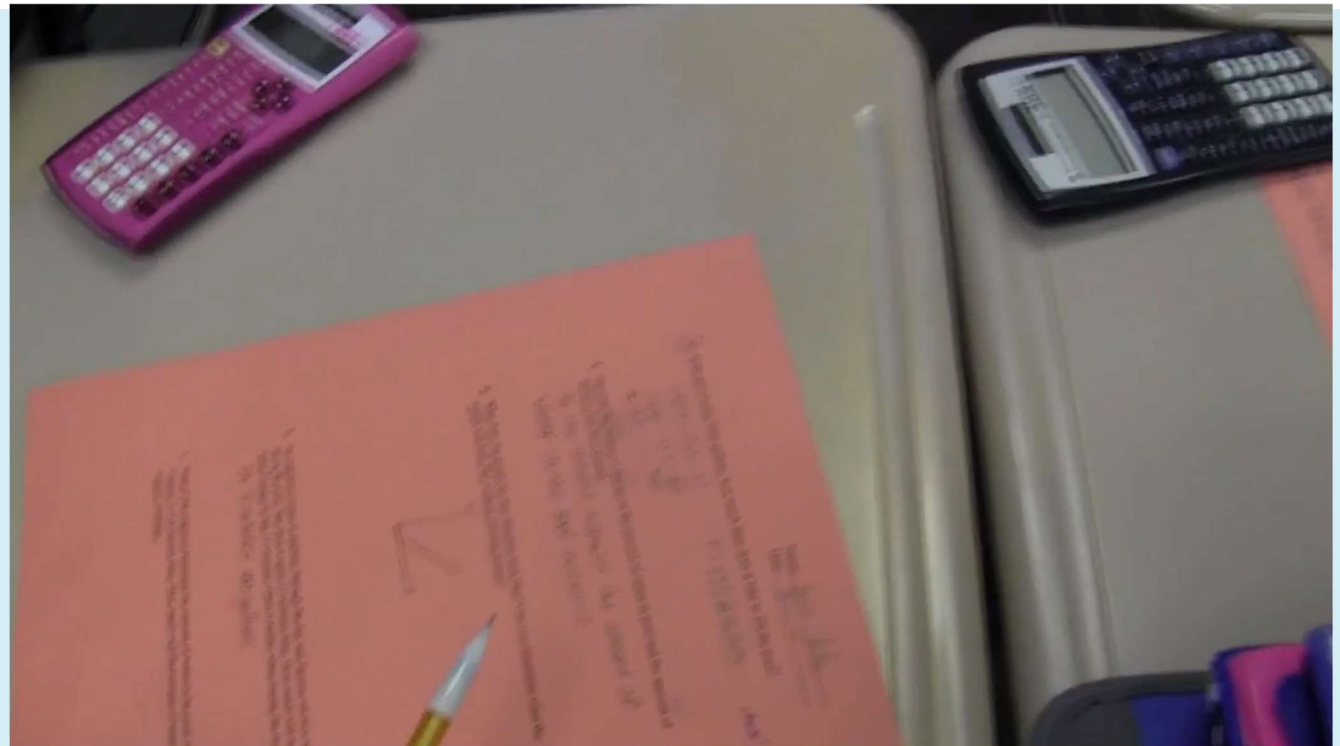
# KATHY



Kathy's initial graph  
for  $w = 12(t - 5)$



Second attempt, 'starting' at 12 gallons, 6 minutes



# QUESTIONS/SHARING

- What does DI look like in your classrooms?
- What questions do you have for Marie and Amy about their experience?
- What questions do you have for our team?

<http://www.indiana.edu/~idream/>

THANK  
YOU