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Differentiating Mathematics Instruction with Middle School Students

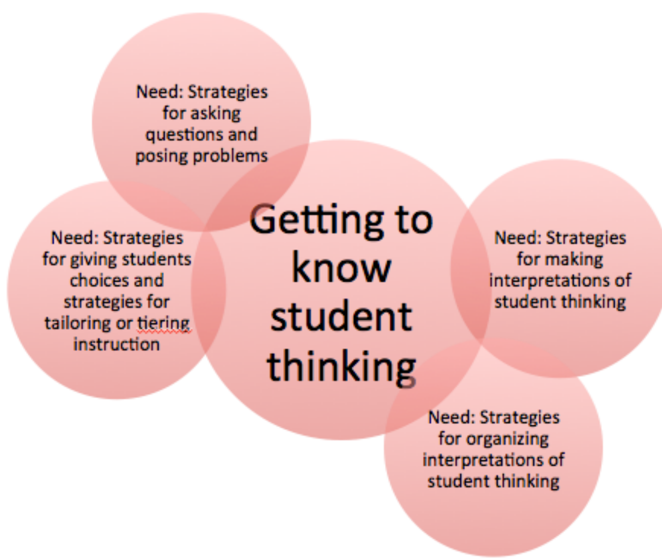
Corn Stalk Tomato Plant Heights Problem. A corn stalk and tomato plant are growing in the garden, each of unknown height. The height of the corn stalk is 5 times the height of the tomato plant.

a) Draw a picture of this situation and describe what your picture represents.
b) Write an equation for this situation that relates the two heights. Explain what your equation means in terms of your picture.
c) Can you write another, different equation that relates the two heights? Explain what your equation means in terms of your picture.
d) If you wrote an equation using division, can you write it with multiplication? Explain what your new equation means in terms of your picture.

In episode 11, the teacher had posted students' pictures and stated, "So in all cases, people were showing that five tomato plant heights fit into a corn stalk height." MC2 student Tim_7 interjected, "Five tomatoes equals *approximately* the corn stalk height." An MC3 student, $Gabriel_8$, countered, "we may not know the actual value, but we do know that it's five times." Despite different views from $Gabriel_8$ and two other students, Tim_7 insisted: "You don't know the corn stalk height, you don't know the tomato plant height. So you don't know anything." Tim_7's view seemed to be that if both heights were unknown, the multiplicative relationship between them would necessarily be uncertain as well. However, once the heights became known, the multiplicative relationship would also become certain. Tim_7 held to this view in subsequent episodes and in his follow-up interview. Other MC2 students did not necessarily hold Tim_7's view explicitly, but in the discussion MC2 student $Lucy_8$ said she understood it, and in further analysis across 3 experiments we found that most MC2 students had difficultly structuring multiplicatively-related unknowns. (We have a paper in preparation about this phenomenon.)

During episode 11 some students worked on a similar problem in which one unknown height was ¼ of another unknown height. In episode 12, the teacher held a discussion about the question in part (d) for this problem. During that discussion MC2 student $Connor_7$ said, "So like if you were to divide the smaller height by 1/4 it'd be like dividing it by 4, so then it'd equal the smaller height." $Lucy_7$ and Tim_7 agreed. In contrast, $Gabriel_8$ and another MC3 student Martin seemed to think it was obvious that the smaller height divided by ¼ was the same as multiplying that height by 4. Yet none of the students knew how to "see" the smaller height divided by ¼ in the picture of the two heights.

What is at the heart of differentiating instruction?



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