The Construction of Quantitative Reciprocal Reasoning

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Purpose

- Show how students who have interiorized three levels of units constructed **quantitative** reciprocal reasoning.
  - I refer to these students as students who have constructed the third multiplicative concept, or MC3 students.

- Comment on a learning trajectory for quantitative reciprocal reasoning for MC3 students.
Quantitative Reciprocal Reasoning

- If A is 2/5 of B, then...
- B is 5/2 of A because 1/5 of B is ½ of A, so 5/5 of B has to be the same as 5/2 of A.
- **NOT:** B is 5/2 of A because you use the reciprocal.
Two Studies

- Study 1: Interview study with 18 middle and high school students, 6 of which were MC3 students.
  - 3 constructed quantitative reciprocal reasoning;
  - Interviewer learned about how to support construction of reciprocal reasoning in interaction.

- Study 2: Design experiments with 18 middle school students (so far), 6 of which were/are MC3 students.
  - All 6 constructed quantitative reciprocal reasoning.
A Quantitative Approach

- Fractions as quantities

- Unknowns as potential measurements of quantities
Tools for Modeling Students' Thinking and Learning

- Operations and schemes
  - Partitioning, iterating, disembedding
  - Iterative fraction scheme: fractional numbers (Steffe & Olive, 2010)

- Accommodation
  - Horizontal and vertical learning
**Concept:** interiorized result of a scheme.

**Composite unit:** a unit of units.

**Units coordination:** distribute the units of one composite unit across the elements of another composite unit.
MC3 Students

- Can take three levels of units as given and flexibly switch between three-levels-of-units structures.
- Maintain views of 35 as a unit of 7 units each containing 5.
- Can construct an iterative fraction scheme.
Iterative Fraction Scheme

Unit Bar

seven-fifths

five-fifths

one-fifth
Possible Outcome of Making Models: Learning Trajectory

- Based on work of Clements & Sarama, 2009; Simon, 1995; Steffe, 2004:
  - Initial model of students' current schemes, operations, and concepts
  - Account of observable changes in those ways of operating
  - Account of mathematical interactions involved in the changes
Fern-Sunflower Problem

*Fern-Sunflower Problem:* A fern and sunflower are growing in the garden, each of unknown height. The height of the sunflower is $\frac{3}{5}$ the height of the fern.

- Draw a picture of this situation and describe what your picture represents.
- Write an equation for this situation that relates the two heights. Explain your equation in terms of your picture.
- Can you write another, different equation that relates the two heights? Explain this equation in terms of your picture.
Martin (7th) and Gabriel (8th)
Martin’s work on JavaBars

- Fern = x
- Sunflower = y

\[ \frac{3}{5}x = \frac{2}{3}y \]

- Fern = x
- Sunflower = y

- \( \frac{1}{5} \) of x = \( \frac{1}{3} \) of y

- \( \frac{3}{5}x = \frac{2}{3}y \)

- Switch between referent unit.
Martin, 7th grade: "... but instead ... the sunflower is the main piece and that’s three parts and this [fern's height] is five and this is one that’s one whole bar plus two fifths so, and that would be five-thirds..."

Andrea, 7th grade: "It’s looking at it in the height of the sunflower, not the height of the fern."

Hector, 10th grade: "... if [the sunflower's height] can be considered a whole, it can be considered a whole but that’s just to itself, so [the fern's height] is more than the whole. So it’s more than one hundred percent, or an improper fraction."
Toward a Learning Trajectory for QRR

- The construction of QRR involves an accommodation in one's iterative fraction scheme, at least.

- What features of students' interactions are important?
  - Focus on quantitative basis/meaning for writing equations.
  - Moving back and forth between algebraic notation and representation of the quantitative situation in drawings.
  - Highlighting moves the teacher-researcher sees as productive, e.g., determining that one part of a height can be named in two ways, depending on referent unit.
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IDR²eAM stands for Investigating Differentiated Instruction and Relationships between Rational Number Knowledge and Algebraic Reasoning in Middle School.