



Investigating Differentiated Mathematics Instruction in Middle School

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IDR²eAM

- Investigating Differentiated Instruction and Relationships between Rational Number Knowledge and Algebraic Reasoning in Middle School
- Differentiating instruction = Proactively tailoring instruction to students' different needs, such as students' readiness and cognitive abilities, interests, and learning profiles and backgrounds (Tomlinson, 2005)
- *Purpose of IDR²eAM*: To investigate how to differentiate mathematics instruction in middle school for students with different key cognitive characteristics.

Years 1-2

- After school math class for nine 7th and 8th grade students with diverse cognitive characteristics
 - Occurs each semester (4 classes total over 2 years)
 - 9 weeks, 18 sessions
 - Video-recorded with 3 cameras and Screenflow software
- Selection of students based on classroom observations, initial interview, math worksheet

Years 3-5

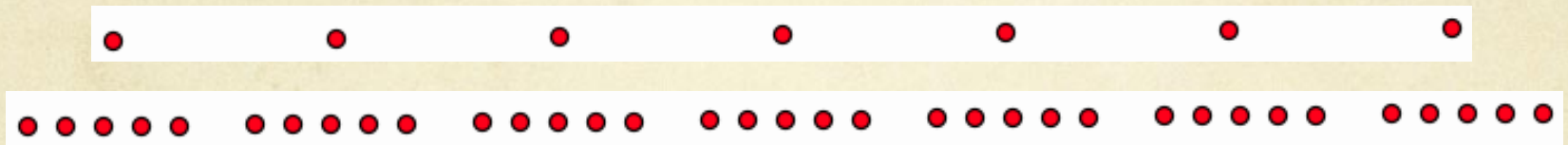
- **Year 3:** Form a study group with about 15 middle school classroom teachers in Indiana to explore how to differentiate math instruction in whole classrooms.
- **Years 4-5:** Co-teach with classroom teachers in classroom experiments to explore differentiated instruction in topics related to rational numbers and algebraic reasoning.

Questions Under Investigation

- (1) How does differentiating mathematics instruction function with middle school students?
- (2) How do students with different key cognitive characteristics use their rational number knowledge to develop algebraic reasoning, and vice versa?
- (3) How does differentiated instruction impact students and teachers, both cognitively and affectively?
- (4) How do teachers develop understanding of and skill at differentiating mathematics instruction for middle school students with different key cognitive characteristics?

Key Cognitive Characteristic: Students' Multiplicative Concepts

- *Concept*: A way of thinking that a student can take as given and read into a situation, prior to acting.
- *Composite unit*: a unit of units
- *Units coordination*: distribute the units of one composite unit across the elements of another composite unit



- First multiplicative concept (MC1 students)

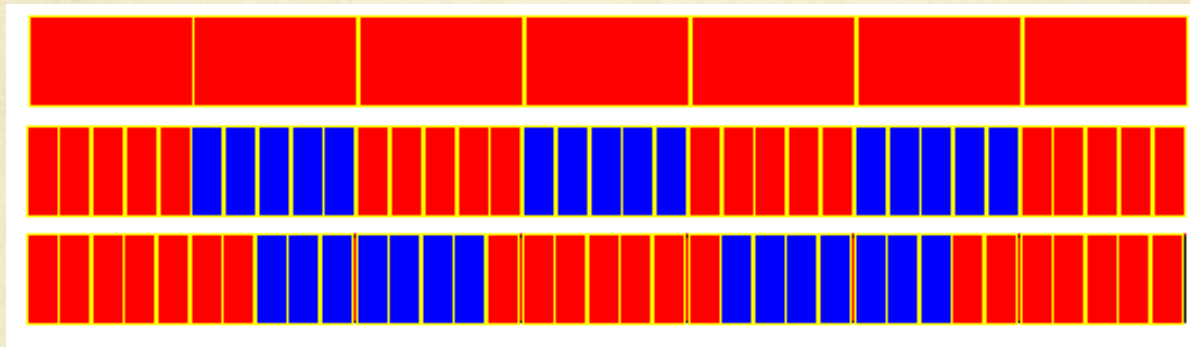
Second Multiplicative Concept (MC2 students)

- Can anticipate the coordination of two levels of units prior to operating
- Can produce three levels of units in activity



Third Multiplicative Concept (MC3 students)

- Can take three levels of units as given and flexibly switch between three-levels-of-units structures



Significance...

- Steffe (2007): 50-70% of incoming sixth grade students are MC2 and MC3 students.
 - ✓ This may imply that roughly $1/3$ of incoming middle school students are operating with each of these three multiplicative concepts.
 - ✓ Advancing to a new multiplicative concept requires vertical learning and can take up to 2 years (Steffe & Cobb, 1988)

Algebra from a Quantitative Perspective

- Unknowns are potential measurements of quantities.



- Thinking of a quantitative unknown—say a distance—requires being able to imagine a unit of units.

Two Goals of IDR²eAM

- Tailor instruction to students' needs:
 - Find out about differences in student thinking that can be a basis for differentiating instruction.
 - Create situations that allow students to learn at their level.
- Develop cohesive classroom community.

Classroom Set-Up

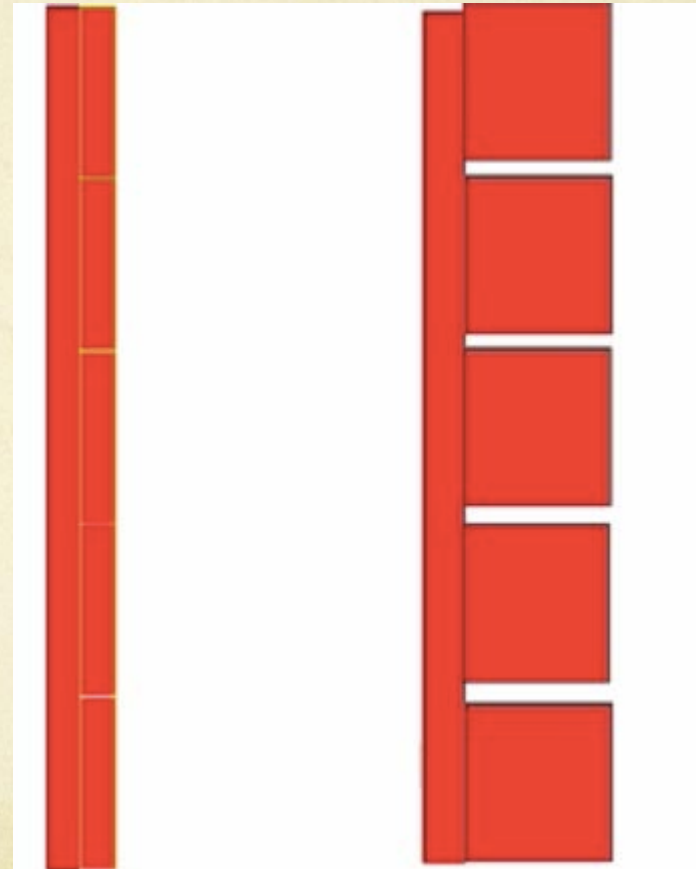


"Approximate" Multiplicative Relationships

There is a tomato plant and stalk of corn growing in the garden, each of unknown height.

The height of the stalk of corn is 5 times the height of the tomato plant.

Draw a picture of this situation and describe what your picture represents.

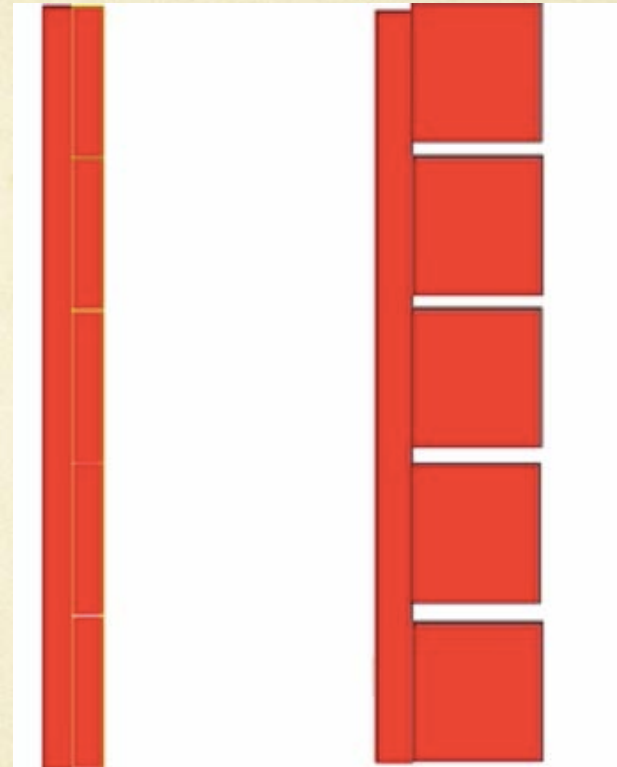


Clip #1: Tim & Gabriel



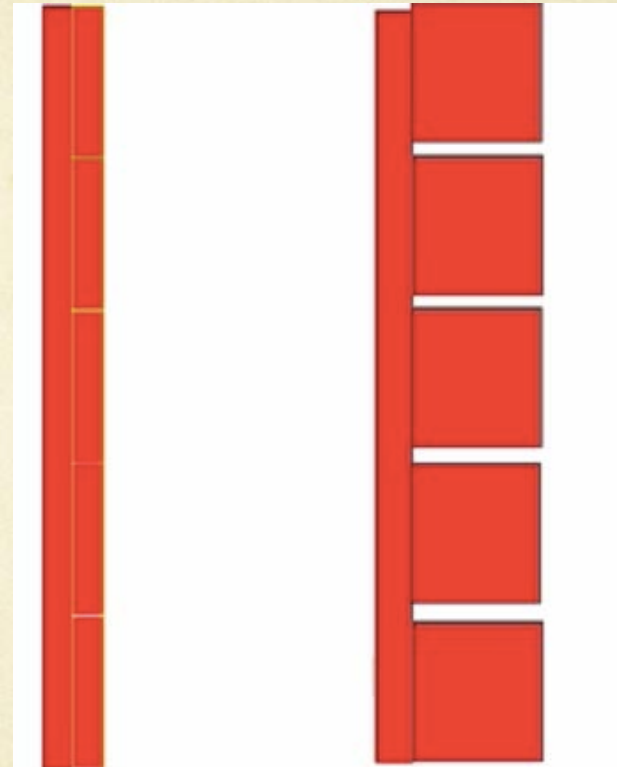
Conjectures about “Approximate”

- Relationships between unknowns for Tim seem to be temporary and approximate.
- Once quantities are known, they can have precise relationships.
- Indefiniteness of quantities implies indefiniteness of relationships.



Implications?

- Say the tomato plant height is x .
- What meaning does $5x$ have for Tim?
- If $5x$ is approximate, how can he get back from $5x$ to x ?
- How can Tim (or other MC2 students) operate meaningfully on $5x$?

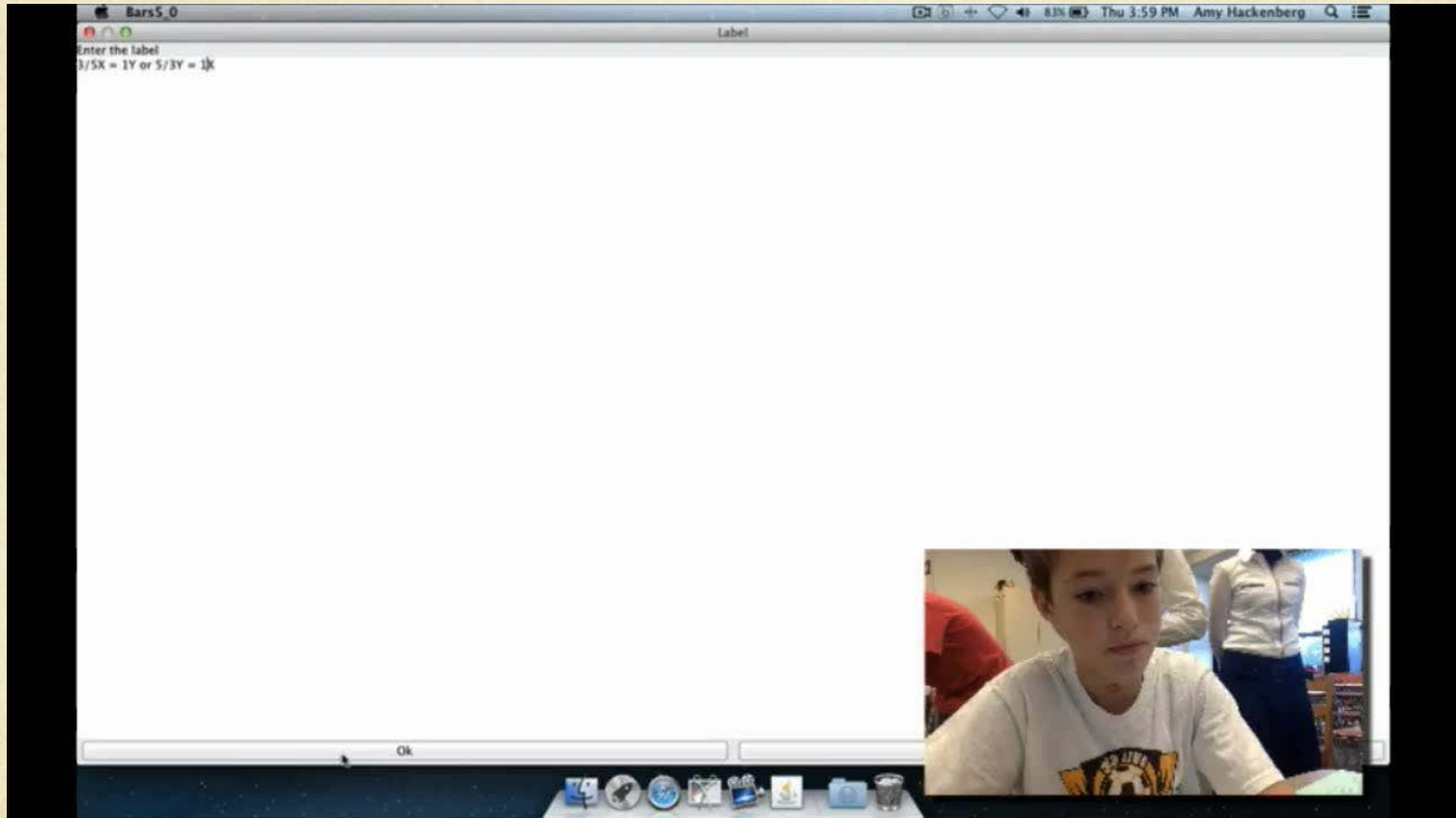


Reciprocal Reasoning

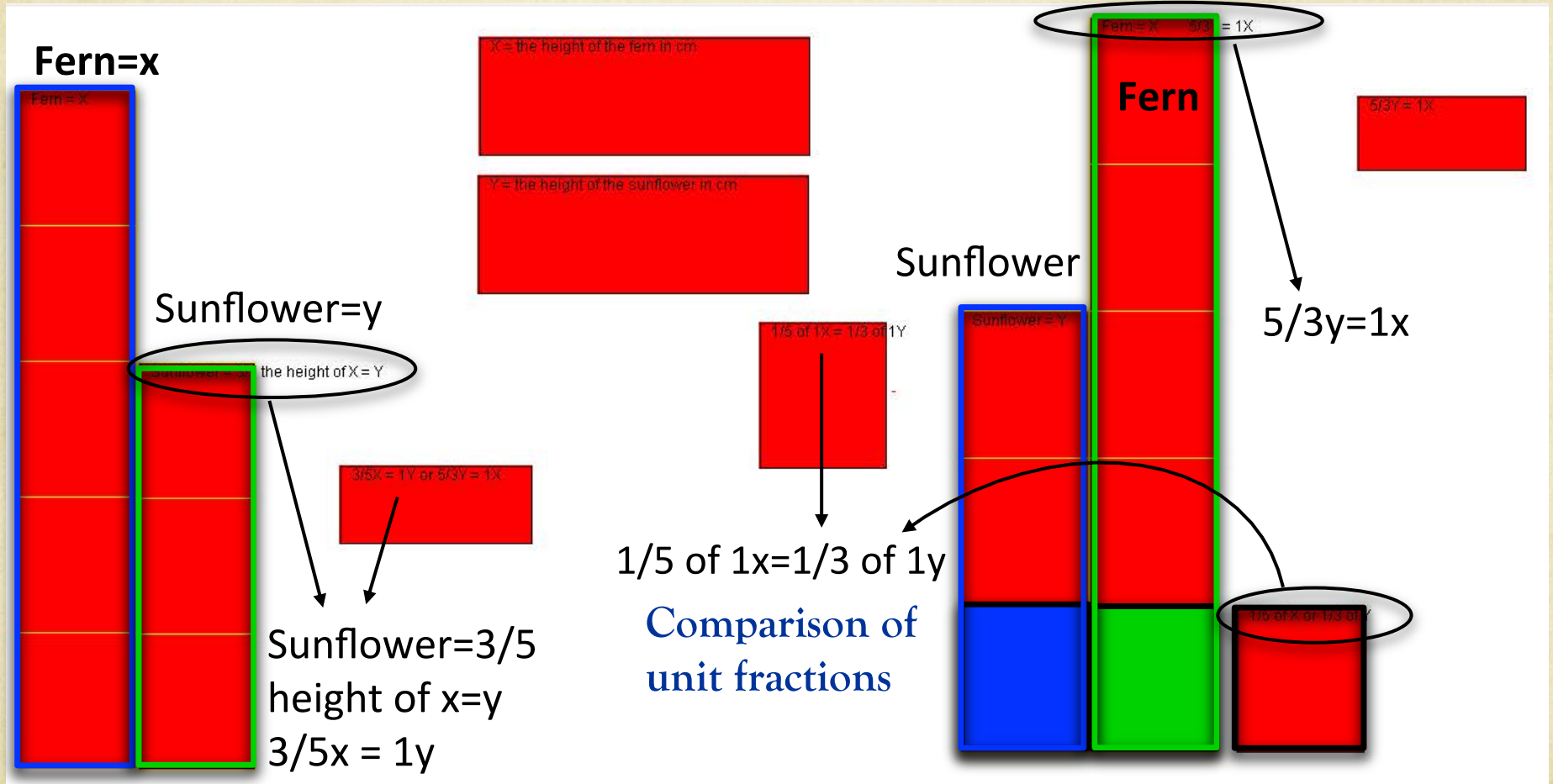
Fern-Sunflower Problem: A fern and sunflower are growing in the garden, each of unknown height. The height of the sunflower is $\frac{3}{5}$ the height of the fern.

- Draw a picture of this situation and describe what your picture represents
- Write an equation for this situation that relates the two heights. **Explain what your equation means in terms of your picture.**
- Can you write another, different equation that relates the two heights? **Explain what your equation means in terms of your picture.**
- If you wrote an equation using division, can you write it with multiplication? **Explain what your new equation means in terms of your picture.**

Clip #2: Martin, Gabriel & Samantha

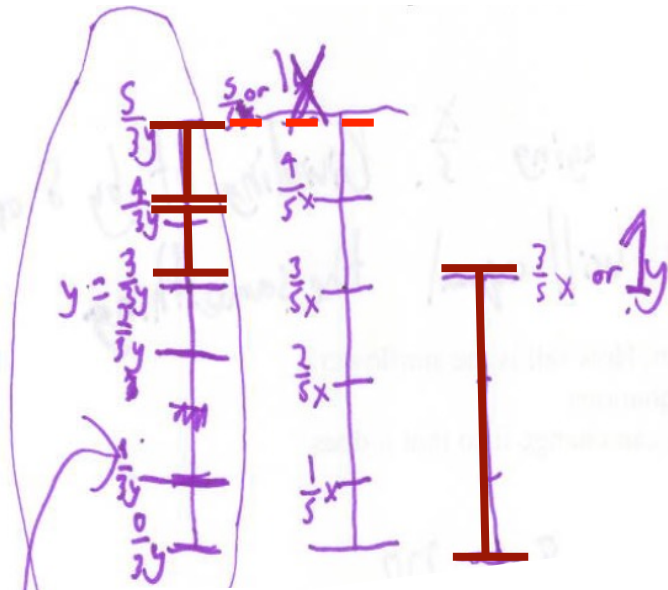


Martin's work on JavaBars



switch between referent unit

Gabriel's written work



$y = \#$ of cm the ~~height~~ ~~is~~ lower is
 $x = \#$ of cm the form is

$\frac{3}{5}$ of x is y , so splitting x into fifths and taking $\frac{3}{5}$ is y .

$$\frac{5}{3}y = x$$

Multiplying y by the ~~reciprocal~~ reciprocal of $\frac{3}{5}$ (which is $\frac{5}{3}$) will equal x . Divide y into $\frac{4}{3}$ thirds and add two extra thirds. You will get the same height as x !

Final Notes...

- We are now conducting our second design experiment, in spring 2014!

THANK YOU!

From all of us working on the IDR²eAM Project

References for DI

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- Tomlinson's website: <http://www.caroltomlinson.com/>
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