Investigating Differentiated Mathematics Instruction in Middle School

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IDR²eAM

- Investigating Differentiated Instruction and Relationships between Rational Number Knowledge and Algebraic Reasoning in Middle School

Differentiating instruction = Proactively tailoring instruction to students' different needs, such as students' readiness and cognitive abilities, interests, and learning profiles and backgrounds (Tomlinson, 2005)

Purpose of IDR²eAM: To investigate how to differentiate mathematics instruction in middle school for students with different key cognitive characteristics.
Years 1-2

- After school math class for nine 7th and 8th grade students with diverse cognitive characteristics
  - Occurs each semester (4 classes total over 2 years)
  - 9 weeks, 18 sessions
  - Video-recorded with 3 cameras and Screenflow software

- Selection of students based on classroom observations, initial interview, math worksheet
Years 3-5

- **Year 3:** Form a study group with about 15 middle school classroom teachers in Indiana to explore how to differentiate math instruction in whole classrooms.

- **Years 4-5:** Co-teach with classroom teachers in classroom experiments to explore differentiated instruction in topics related to rational numbers and algebraic reasoning.
Questions Under Investigation

1. How does differentiating mathematics instruction function with middle school students?

2. How do students with different key cognitive characteristics use their rational number knowledge to develop algebraic reasoning, and vice versa?

3. How does differentiated instruction impact students and teachers, both cognitively and affectively?

4. How do teachers develop understanding of and skill at differentiating mathematics instruction for middle school students with different key cognitive characteristics?
Key Cognitive Characteristic: Students' Multiplicative Concepts

- **Concept:** A way of thinking that a student can take as given and read into a situation, prior to acting.

- **Composite unit:** a unit of units

- **Units coordination:** distribute the units of one composite unit across the elements of another composite unit

- First multiplicative concept (MC1 students)
Second Multiplicative Concept (MC2 students)

- Can anticipate the coordination of two levels of units prior to operating
- Can produce three levels of units in activity
Third Multiplicative Concept (MC3 students)

- Can take three levels of units as given and flexibly switch between three-levels-of-units structures
Significance...

• Steffe (2007): 50-70% of incoming sixth grade students are MC2 and MC3 students.
  ✓ This may imply that roughly 1/3 of incoming middle school students are operating with each of these three multiplicative concepts.
  ✓ Advancing to a new multiplicative concept requires vertical learning and can take up to 2 years (Steffe & Cobb, 1988)
Algebra from a Quantitative Perspective

- Unknowns are potential measurements of quantities.

- Thinking of a quantitative unknown—say a distance—requires being able to imagine a unit of units.
Two Goals of IDR$^2$eAM

- Tailor instruction to students' needs:
  - Find out about differences in student thinking that can be a basis for differentiating instruction.
  - Create situations that allow students to learn at their level.

- Develop cohesive classroom community.
Classroom Set-Up
"Approximate" Multiplicative Relationships

There is a tomato plant and stalk of corn growing in the garden, each of unknown height.

The height of the stalk of corn is 5 times the height of the tomato plant.

Draw a picture of this situation and describe what your picture represents.
Clip #1: Tim & Gabriel
Conjectures about “Approximate”

- Relationships between unknowns for Tim seem to be temporary and approximate.
- Once quantities are known, they can have precise relationships.
- Indefiniteness of quantities implies indefiniteness of relationships.
Implications?

• Say the tomato plant height is $x$.

• What meaning does $5x$ have for Tim?

• If $5x$ is approximate, how can he get back from $5x$ to $x$?

• How can Tim (or other MC2 students) operate meaningfully on $5x$?
Reciprocal Reasoning

Fern-Sunflower Problem: A fern and sunflower are growing in the garden, each of unknown height. The height of the sunflower is 3/5 the height of the fern.

- Draw a picture of this situation and describe what your picture represents

- Write an equation for this situation that relates the two heights. Explain what your equation means in terms of your picture.

- Can you write another, different equation that relates the two heights? Explain what your equation means in terms of your picture.

- If you wrote an equation using division, can you write it with multiplication? Explain what your new equation means in terms of your picture.
Clip #2: Martin, Gabriel & Samantha
Martin's work on JavaBars

Fern = x
Sunflower = y

Comparison of unit fractions

$$\frac{3}{5} \text{ of } x = \frac{1}{3} \text{ of } y$$

Sunflower = $$\frac{3}{5}$$
Height of x = y
$$\frac{3}{5}x = 1y$$

Fern = $$\frac{5}{3}y = 1x$$

Switch between referent unit
Gabriel’s written work

\[
y = \frac{3}{5}y \quad \text{for} \quad x \quad \text{or}\quad \frac{1}{5}y
\]

\[
y = \frac{3}{5}x \quad \text{or} \quad \frac{1}{5}y
\]

\[
y = \frac{5}{3}y = x
\]

Multiplying \( y \) by the reciprocal of \( \frac{3}{5} \) (which is \( \frac{5}{3} \)) will equal \( x \). Divide \( y \) into \( \frac{3}{5} \) thirds and add two extra thirds, you will get the same height as \( x \! \).
Final Notes...

- We are now conducting our second design experiment, in spring 2014!
THANK YOU!
From all of us working on the IDR²eAM Project
References for DI


- [http://differentiationcentral.com/](http://differentiationcentral.com/)