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Purpose:

- Communicate what I mean by learning fractional knowledge from students.
- Show an example of how students' fractional knowledge is involved in working on algebraic problems.
- Indicate that students' fractional knowledge is a structured body of knowledge that I am learning.

Position of my knowing in relation to students' knowing

- Don't assume my mathematical knowledge is sufficient to understand students' mathematical thinking
 - Cannot "escape" my own knowledge
 - Can decenter from it
- Expect that students will view situations differently, but that their ways of knowing
 - Are rational
 - Are a world to themselves, not a stripped down version of my (or other adult) knowledge

Second-order models

- Constellation of constructs that describe and account for another's ways of operating
- Require
 - Extended interactions with students
 - Thinking tools from prior research (helpful)
 - Openness to developing new ways of thinking

Tools...broadly

- I view learning in the context of making accommodations in schemes in on-going interaction in one's experiential world
- Schemes: goal-directed ways of operating that include a perceived situation, activity, and perceived result.

• Accommodations: reorganizations of and modifications in schemes



Steffe & Olive (2010, p. 23)

Tools... more specifically: Partitive fraction scheme

• Mental operations: partitioning, disembedding, and iterating

| U | Unit Bar | | | | | | |
|---|----------|--|--|--|--|--|--|
| Γ | | | | | | | |
| Ī | | | | | | | |

• Meaning for result is part of a whole

| Tools | more specifi | cally: |
|-----------|----------------|--------|
| Iterative | e fraction sch | neme |

• Fractions are whole number multiples of unit fractions

| Unit Bar | | | | | | |
|-------------------------------------|--|--|--|--|--|--|
| seven-fifths, a unit of seven units | | | | | | |
| five-fifths, a unit of five units | | | | | | |
| one-fifth | | | | | | |
| | | | | | | |
| | | | | | | |

What is a fraction?

- Who are we talking about?
- A range of ways of thinking I have developed to describe and account for others' ways of thinking that fit with, and may depart from, criteria developed by people who study the ontogenesis of fractional knowledge:
 - Piaget, Inhelder, & Szeminska (1960): 7 criteria for operational subdivision
 - Steffe and Olive (2010): 4 levels of fragmenting
- The problem with definitions...

Methods in current project

- Design experiments with 9 middle school students who have constructed different fractional schemes
 - 18 1-hour episodes, twice per week
- Project purposes:
 - Study how to differentiate instruction for cognitively diverse middle school students
 - Study how students' rational number knowledge and algebraic reasoning are related

Participants, fall 2013

| School math course | Part-whole or partitive unit fraction scheme | Partitive fraction scheme | Iterative fraction scheme |
|---|---|--------------------------------------|---------------------------------|
| 7 th grade math | Paige, Tim | Connor, Andrea, Diamond | |
| Adv. 7 th grade math | | | Martin, Stephanie |
| 8 th grade pre-alg. | Lucy | | |
| Alg. (7 th & 8 th) | | | Gabriel (8 th) |
| Totals | 3 | 3 | 3 |

Multiplicative relationships between unknowns

Heights of Dogs Problem. Rover and Spot are dogs. Rover's height is 2/5 the height of Spot. We don't know either height, so they are both unknowns.

- Draw a picture of this situation and describe what your picture represents.
- Write an equation for this situation that relates the two heights. Explain what your equation means in terms of your picture.
- Can you write another, different equation that relates two heights? Explain what your equation means in terms of your picture.

Connor₇'s comments on 2/5





Connor₇'s final picture

for the Heights of Dogs Problem

Martin₇'s picture and notation for 3/5 relationship

- For a problem involving a 3/5 relationship: y = 3/5x x = 5/3y
- Initial reason for x = 5/3y: "It's the reciprocal."



Martin₇'s reciprocal reasoning with quantitative unknowns



Martin's picture for Pine Tree Problem in Follow-up Interview

Pine Tree Heights Problem. Steven and Lia each are growing a pine tree. The height of each of their trees is unknown. The height of Lia's plant is 2/5 the height of Steven's plant.



Martin₇'s comments on 2/5 of an unknown





Martin₇'s notation

for the Pine Tree Heights Problem

What I hope you will take away...

- A sense that students' fractional knowledge something that I as a researcher make—is structured:
- It is my knowledge because all I have is my ways of perceiving and conceiving!
- It is not my knowledge, in the sense that students make contributions to shape it that are independent of my ways of thinking and are not striped down versions of my thinking:
 - To understand I have to learn new ways of thinking that are legitimate, mathematical ways of thinking.
- Second-order models are truly co-constructed with students (Steffe & Olive, 2010).

Thank you!

- With BIG thanks to the IDR²eAM project team: Fetiye Aydeniz, Rebecca Borowski, Mark Creager, Ayfer Eker, Robin Jones, Musa Sadak, Serife Sevis, Pai Suksak, Ryan Timmons
- What IDR²eAM stands for: Investigating
 Differentiated Instruction and Relationships
 between Rational Number Knowledge and
 Algebraic Reasoning in Middle School
- o http://www.indiana.edu/~idream/



Lucy₈'s final picture